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ADVANCED DURABILITY ANALYSIS.  
VOLUME I.- ANALYTICAL METHODS.

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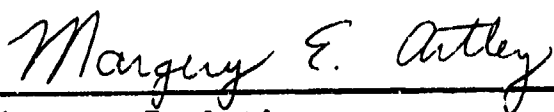
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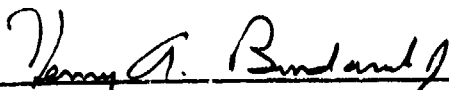


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| 19. ABSTRACT (Continue on reverse if necessary and identify by block number)<br>Advanced durability analysis "design tools" have been developed for metallic aircraft structures. These tools can be used to evaluate durability design requirements for functional impairments due to (1) excessive cracking and (2) fuel leakage/ligament breakage. The methodology accounts for the initial fatigue quality variation of structural details, the crack growth accumulation for a population of structural details under specified design conditions and structural properties. Step-by-step procedures are provided. This volume is limited to the analytical methods, technical aspects, concepts and philosophy for the durability analysis of metallic aircraft structures.<br><br>The methodology reflects a probabilistic approach, a fracture mechanics philosophy and both deterministic and stochastic crack growth methods. It can be used to predict the probability of crack exceedance at any service time and/or the cumulative distribution of the time- |   |   |                                   |                     |
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18. (continued) probability of crack exceedance, cumulative distribution of TICI.

19. (continued) to-crack initiation (TICI) at any crack size. The methodology applies to the small crack size range associated with excessive cracking (e.g.,  $<0.05''$ ) and to large through-the-thickness cracks (e.g.,  $0.50''-0.75''$ ) associated with fuel leakage/ligament breakage.

No matter what form, location or combination the as-manufactured flaws may have in fastener holes (e.g., scratches, burrs, microscopic imperfections, etc.) or whatever the source of fatigue cracking may be, a practical method of representing the reality of the as-manufactured condition is needed for durability analysis. This is taken care of by the equivalent initial flaw concept.

Initial fatigue quality of a structural detail (e.g., fastener holes, cutouts, fillets, lugs, etc.) is represented by an equivalent initial flaw size distribution. An equivalent initial flaw (EIFS) is an artificial crack size which results in an actual crack size at an actual point in time when the initial flaw is grown forward. It is determined by back-extrapolating fractographic results. It has the following characteristics:

(1) An EIFS is an artificial crack assumed to represent the initial fatigue quality of a structural detail in the as-manufactured condition whatever the source of fatigue cracking may be, (2) it has no direct relationship to actual initial flaws in fastener holes such as scratches, burrs, microdefects, etc., and it cannot be verified by NDI, (3) it has a universal crack shape in which the crack size is measured in the direction of crack propagation, (4) EIFSs are in a fracture mechanics format but they are not based on linear elastic fracture mechanics (LEFM) principles and are not subject to such laws or limitations such as the "short crack effect," (5) it depends on the fractographic data used, the fractographic crack size range used for the back-extrapolation and the crack growth rate model used, (6) it must be grown forward in a manner consistent with the basis for the EIFS, (7) EIFSs are not unique - a different set is obtained for each crack growth law used for the back-extrapolation, and (8) EIFSs are not necessarily comparable with minimum crack sizes obtained from fractographic results from tear-down inspections.

Recommendations for durability analysis are as follows: (1) define the equivalent initial flaw size distribution (EIFSD) using fractographic data in the small crack size region (e.g.,  $0.01''-0.05''$ ), (2) use fractographic data pooling procedure and statistical scaling technique to estimate the EIFSD parameters in a "global sense" for a "single hole population" basis (3) use the deterministic crack growth approach (DCGA) in the small crack size region and (4) use the two-segment deterministic-stochastic crack growth approach (DCGA-SCGA) for applications in the large crack size region (e.g.,  $0.50''-0.75''$ ); the two-segment deterministic crack growth approach (DCGA-DCGA) is also reasonable but it is slightly less conservative than the DCGA-SCGA.

Procedures have been developed for defining initial fatigue quality. These procedures could be used to standardize the way initial flaw sizes are determined from fractographic data. A better understanding of initial flaw sizes (i.e., what they are and limitations) has been developed. For consistent durability analysis predictions, equivalent initial flaws must be used in the same context for which they were defined. This means that equivalent initial flaws must be grown forward in the same manner the EIFSs were established by back-extrapolating fractographic results.



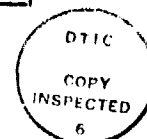
## FOREWORD

This report was prepared by General Dynamics, Fort Worth Division, under the "Advanced Durability Analysis" program (Air Force Contract F33615-84-C-3208) for the Air Force Wright Aeronautical Laboratories (AFWAL/FIBEC). Margery E. Artley was the Air Force Project Engineer; Dr. John W. Lincoln of ASD/ENFS and James L. Rudd of AFWAL/FIBEC were technical advisors. Dr. S. D. Manning of the General Dynamics' Structures Technology Staff was the program manager and co-principal investigator along with Dr. J. N. Yang of United Analysis Incorporated (Vienna, VA).

This report (Vol. I) documents the analytical methods and theoretical aspects developed under the "Advanced Durability Analysis" program. Other volumes for this program are as follows:

- o Volume II - Analytical Predictions, Test Results, and Analytical/Experimental Correlations
- o Volume III - Fractographic Test Data
- o Volume IV - Executive Summary
- o Volume V - Durability Analysis Software User's Guide

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## SECTION I

### INTRODUCTION

#### 1.1 OBJECTIVE AND SCOPE

This program had three objectives: (1) recommend improvements to the current Air Force durability design requirements (MIL-A8866B)[1], (2) develop a probabilistic durability analysis method capable of predicting the durability of advanced metallic aircraft structures for such functional impairment definitions as excessive cracking, fuel leakage and ligament breakage, and (3) update the current Air Force Durability Design Handbook (AFWAL-TR-83-3027)[2]. Fatigue cracking is the form of degradation to be considered.

This report documents the advanced durability analysis methodology for metallic aircraft structures developed under the Phase I effort. Phase I included three tasks as follows: (1) Task I - refine, optimize and increase confidence in the method used to determine the equivalent initial flaw size distribution (EIFSD) for clearance-fit fastener holes, (2) Task II - refine procedure and guidelines for predicting crack exceedance in the small crack size region (e.g.,  $<0.10$ "), and (3) Task III - extend the prototype durability analysis methodology to large crack sizes (e.g.,  $0.5$ " -  $0.75$ " through-the-thickness) associated with functional impairment due to fuel leakage and ligament breakage.

This volume is limited to the analytical methods, theoretical aspects, concepts and philosophy for advanced durability analysis. The methodology presented in this report is evaluated and justified for durability analyses in Volume II [3]. Three other volumes for the program are as follows: (1) Volume III - Fractographic Test Data [4], (2) Volume IV - Executive Summary [5], and (3) Volume V - Durability Analysis

Software User's Guide [6]. The Durability Design Handbook [7] has also been updated for the advancements of this program.

## 1.2 BACKGROUND

The original Air Force durability design requirements [1] are both brief and vague. New requirements have recently been proposed [8], including assumptions for the initial fatigue quality (IFQ) of a fastener hole. A 0.01" initial quarter-circular corner crack, assumed to exist in a fastener hole, is intended to represent the IFQ of the structure. However, the same initial crack size is used to represent the IFQ irrespective of the material selected, the hole drilling procedures used, and the manufacturing and assembly techniques used. A more realistic representation of the IFQ of structural details is needed, including standard methods for defining IFQ.

A durability analysis methodology [2,9-17] has been developed for satisfying the durability design requirements of MIL-A-8866B. This methodology accounts for IFQ variation, fatigue crack growth accumulation in a population of structural details, structural properties and durability analysis conditions (e.g., load spectra, stress level, % bolt load transfer, etc.). A statistical distribution of equivalent initial flaw size (EIFS) is used to represent the IFQ of structural details (e.g., fastener holes, fillets, cutouts, lugs, etc.). The methodology reflects a fracture mechanics philosophy, a probabilistic format and deterministic crack growth. The equivalent initial flaw size distribution (EIFSD) is grown forward, using the applicable durability design conditions, to a selected service time using a deterministic crack growth approach. Additional references on the methodology are available [18-26].

The prototype durability analysis methodology uses an "engineering approach" to represent the IFQ of structural details. IFQ is represented by an equivalent initial flaw size distribution (EIFSD) function. Engineering principles rather than mechanistic-based theories for microstructural behavior are used. This approach is justified for durability design applications.

An empirical-based crack growth rate model and fractographic results are used to define the EIFSD. Equivalent initial flaws are determined by back-extrapolating fractographic results to time zero. Such flaws are artificial rather than actual initial flaws in a fastener hole, such as a scratch, a burr, or a void. Since EIFSs cannot be verified by NDI, indirect methods are used to justify their use for durability analyses.

The methodology uses an EIFSD function to represent the IFQ. However, the parameters for describing these distributions have not been optimized. The sensitivity of the parameters with respect to the material, geometry, crack size range, maximum stress level, amount of load transfer, and load spectra has not been established. Such an optimization and sensitivity study is needed to adequately represent the initial fatigue quality of aircraft structure.

Once the EIFSD has been determined, a service crack growth master curve is used to predict the probability of crack exceedance,  $p(i, \tau)$ , for structural details at future points in time. However, the methods for predicting  $p(i, \tau)$  have not been optimized. Such an optimization study is needed to accurately predict the probability that a crack will exceed a specific size at the point in time of interest.

Fractographic data are needed to determine suitable EIFSDs for durability analysis. However, design guidelines

do not currently exist for specifying the specimen geometries, number of holes per specimen, number of maximum stress levels, number of load transfer levels, number of load spectra, and crack size ranges which are needed to adequately represent the initial fatigue quality of aircraft structure. Similarly, guidelines do not exist for making accurate  $p(i,T)$  predictions.

The durability analysis methodology was based on a durability definition involving the rework of fastener holes (i.e., the reaming out of fastener holes to the next nominal hole size). Such a definition involves small crack sizes (i.e., less than 0.1"). However, durability can also include definitions of functional impairment such as fuel leakage and ligament breakage. Such definitions normally involve crack sizes greater than 0.1". Hence, the durability analysis methodology needs to be extended to include large crack sizes. Such an extension for large crack sizes indicates that some of the small-crack-size assumptions used for the durability analysis methodology are invalid and must be replaced with more appropriate formulations.

## SECTION II

### DURABILITY ANALYSIS METHODS FOR SMALL CRACK SIZE RANGE

Advanced durability analysis methodology for metallic airframes is described and discussed in this section for evaluating durability requirements for functional impairment due to excessive cracking. General principles and procedures are presented for application in the small crack size range. The durability analysis extension covering the large crack size range (e.g., crack sizes associated with functional impairment due to fuel leaks/ligament breakage) is presented in Section III.

A durability analysis methodology for the small crack size region (e.g.,  $<0.10''$ ) was developed under the previous program [2,9-17]. Under the current program several improvements and advancements have been made in the durability analysis methodology for application in the small crack size range. In particular, improved methods, procedures and guidelines have been developed for: (1) defining a suitable equivalent initial flaw size distribution, and (2) making durability analysis predictions for the probability of crack exceedance and cumulative distribution of TTCI. Significant improvements and refinements have been made in the methods for defining the initial fatigue quality of structural details. Methods and software for an IBM compatible PC [6] have been developed for estimating and optimizing the EIFSD parameters. Technical details of the advancements made are described and discussed in this Volume (I). The improved methods and procedures are evaluated and justified in Volume II [3].

In this section the advanced durability analysis methodology for the small crack size range is summarized and de-

tails are given in Appendices A-G. The advanced durability analysis methodology presented in this section is formulated based on the Weibull compatible EIFSD function and the deterministic crack growth approach (DCGA). Formulations for the following EIFSD functions are presented in Appendix D: two-parameter Weibull, lognormal and lognormal compatible.

Five different methods were investigated under this program for estimating the EIFSD parameters: (1) combined least square sums approach (CLSSA), (2) method of moments, (3) homogeneous EIFS approach (HEIFS), (4) upper tail fit and (5) nonlinear approach. Only the CLSSA is presented in this section and the other approaches are described and discussed in Appendix F.

## 2.1 DURABILITY ANALYSIS APPROACH

The durability analysis approach for the small crack size range (e.g., <0.10") is conceptually described in Fig. 1. This approach reflects a probabilistic format with a deterministic crack growth law. The initial fatigue quality of structural details is represented by an equivalent initial flaw size distribution (EIFSD). The durability analysis approach accounts for initial fatigue quality, crack growth accumulation in a population of structural details, load spectra and structural properties. Essential elements, equations and features of the approach are described in the following.

The Weibull compatible distribution function proposed by Yang and Manning [13,18] has been found to be reasonable for representing the EIFS cumulative distribution [2,13,18, 24-30].

$$F_{a(0)}(x) = \exp \left\{ - \left[ \frac{\ln(x_u/x)}{\phi} \right]^a \right\} ; 0 < x \leq x_u \quad (1)$$

$$= 1.0 \quad ; x \geq x_u$$



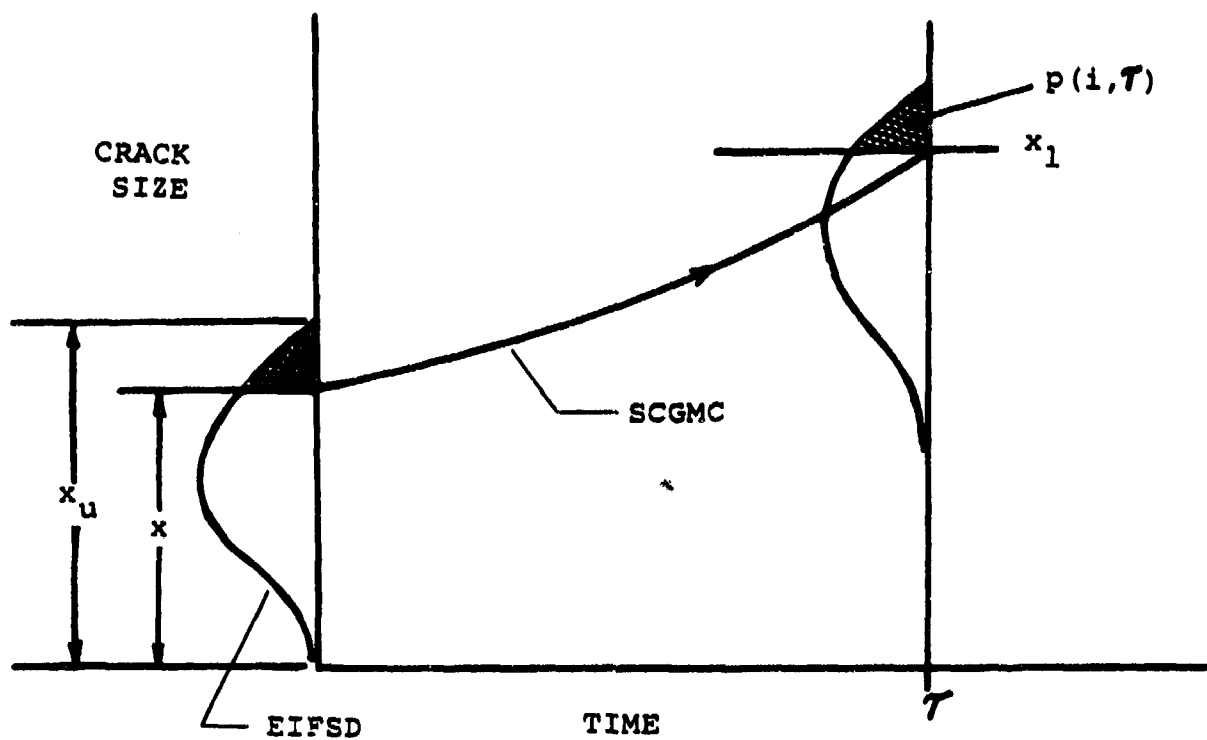


Figure 1. Deterministic Crack Growth Approach (DCGA).

in which  $F_{a(0)}(x) = P[a(0) \leq x]$ ,  $a(0)$  = EIFS = crack size at time  $t = 0$ ,  $x_u$  = EIFS upper bound limit and  $\alpha$  and  $\phi$  are empirical parameters.

A simple deterministic crack growth rate model for the small crack size region designed and proposed by Yang and Manning [13,18], is used

$$da(t)/dt = Q[a(t)]^b \quad (2)$$

where:  $da(t)/dt$  = crack growth rate,  $a(t)$  = crack size at any time  $t$ ;  $Q$  and  $b$  are empirical crack growth rate parameters. Using the special case  $b = 1$  and integrating Eq. 2, one obtains

$$a(t) = a(0) \exp(Qt) \quad (3)$$

When the equivalent initial flaw size takes a sample value  $x$ , i.e.,  $a(0) = x$ , it will grow to the corresponding sample value  $x_1$  at time  $t$ , i.e.,  $a(t) = x_1$ . It follows from Eq. 3 that

$$x_1 = x \exp (Qt) \quad (4)$$

Equation 4 is referred to as the "EIFS Master Curve."

The cumulative distribution of crack size  $a(t)$  at any time  $t$ , denoted by  $F_{a(t)}(x) = P[a(t) \leq x]$ , can be obtained from the Weibull compatible distribution of  $a(0)$  given by Eq. 1 through the transformation of Eq. 3 as follows (Fig. 2).

$$F_{a(t)}(x) = \exp \left\{ - \left[ \frac{\ln(x_u/x) + Qt}{\phi} \right]^\alpha \right\} ; x > 0 \text{ and } t \geq 0 \quad (5)$$

The probability that the crack size  $a(\tau)$  at any service time will exceed any given crack size  $x_1$ , denoted by  $p(i, \tau) =$

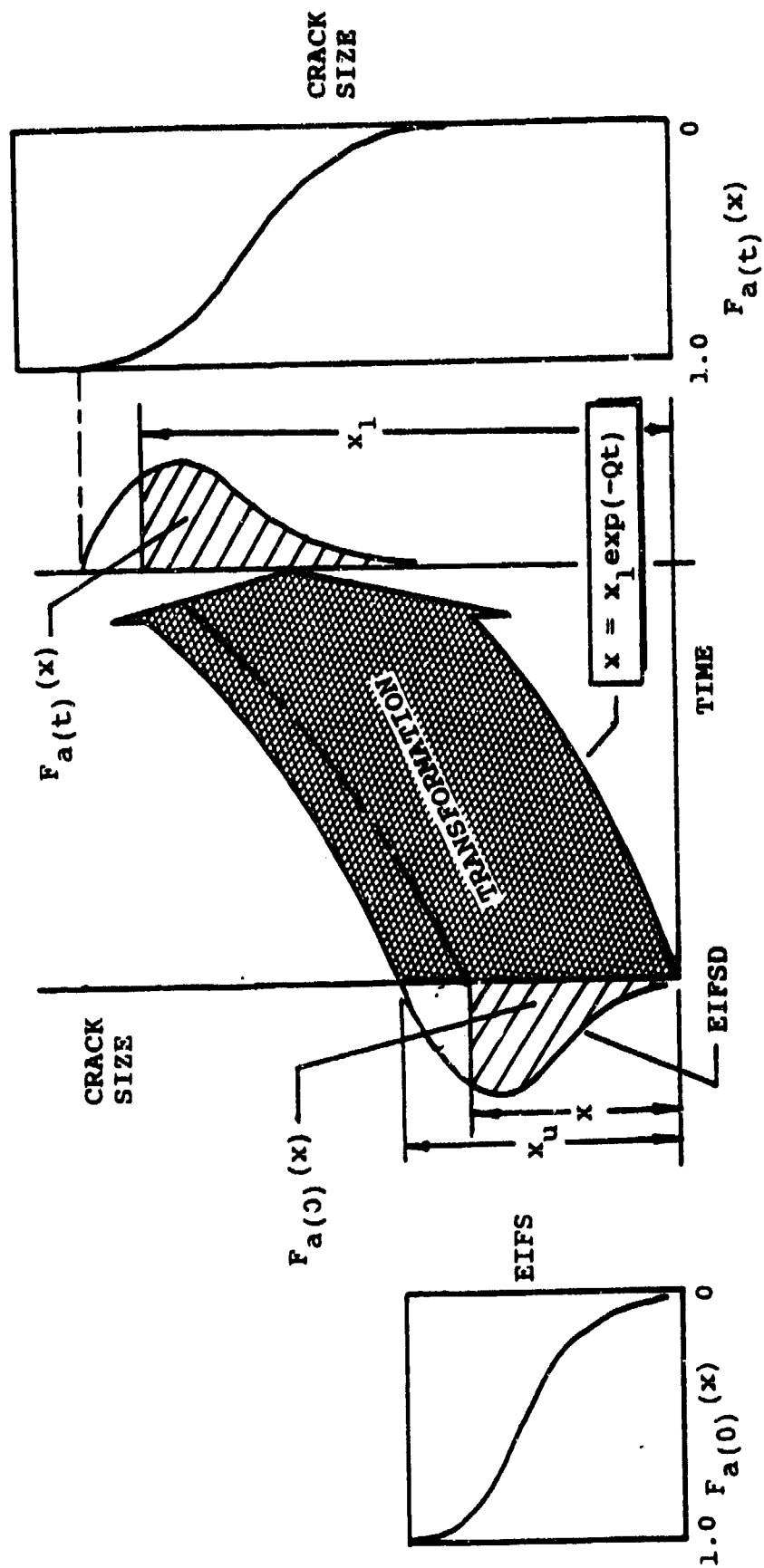


Figure 2. Transformation of the Cumulative Distribution of EIFS,  $F_a(0)(x)$ , to Obtain the Cumulative Distribution of Crack Size,  $F_a(t)(x)$ .

$P[a(\tau) > x_1]$ , is referred to as the crack exceedance probability (Fig. 1).

$$p(i, \tau) = 1 - F_a(\tau)(x_1) = 1 - \exp \left\{ - \left[ \frac{\ln(x_u/x_1) + Q\tau}{\phi} \right]^\alpha \right\} \quad (6)$$

If  $F_T(t)$  denotes the cumulative distribution of time to initiate a given crack size  $x_1$ , referred to as time to crack initiation (T or TICI), then it follows that

$$F_T(t) = P[T \leq t] = 1 - F_{a(t)}(x) = p(i, t) \quad (7)$$

$$= 1 - \exp \left\{ - \left[ \frac{\ln(x_u/x_1) + Qt}{\phi} \right]^\alpha \right\}$$

Hence, the prediction of the cumulative distribution of TICI,  $F_T(t)$ , can be made using Eq. 7. The expression for  $F_T(t)$ , given in Eq. 7, can also be obtained using Eq. 1 through a transformation of Eq. 4 as conceptually described in Fig. 3.

Equations 5-7 are based on the Weibull compatible EIFSD function given in Eq. 1. Similar expressions, based on the two-parameter Weibull, lognormal and lognormal compatible EIFSD functions, are given in Appendix D.

## 2.2 INITIAL FATIGUE QUALITY REPRESENTATION

The initial fatigue quality (IFQ) defines the initially manufactured state of a structural detail or details with respect to initial flaws in a part, component, or airframe prior to service. An equivalent initial flaw size distribution (EIFSD) is used to represent the IFQ of replicate details (e.g., fastener holes, cutouts, lugs, fillets, etc.).

An equivalent initial flaw (EIFS) is an artificial crack size which results in an actual crack size at an actual point in time when the initial flaw is grown forward. It is determined by back-extrapolating fractographic results. It has

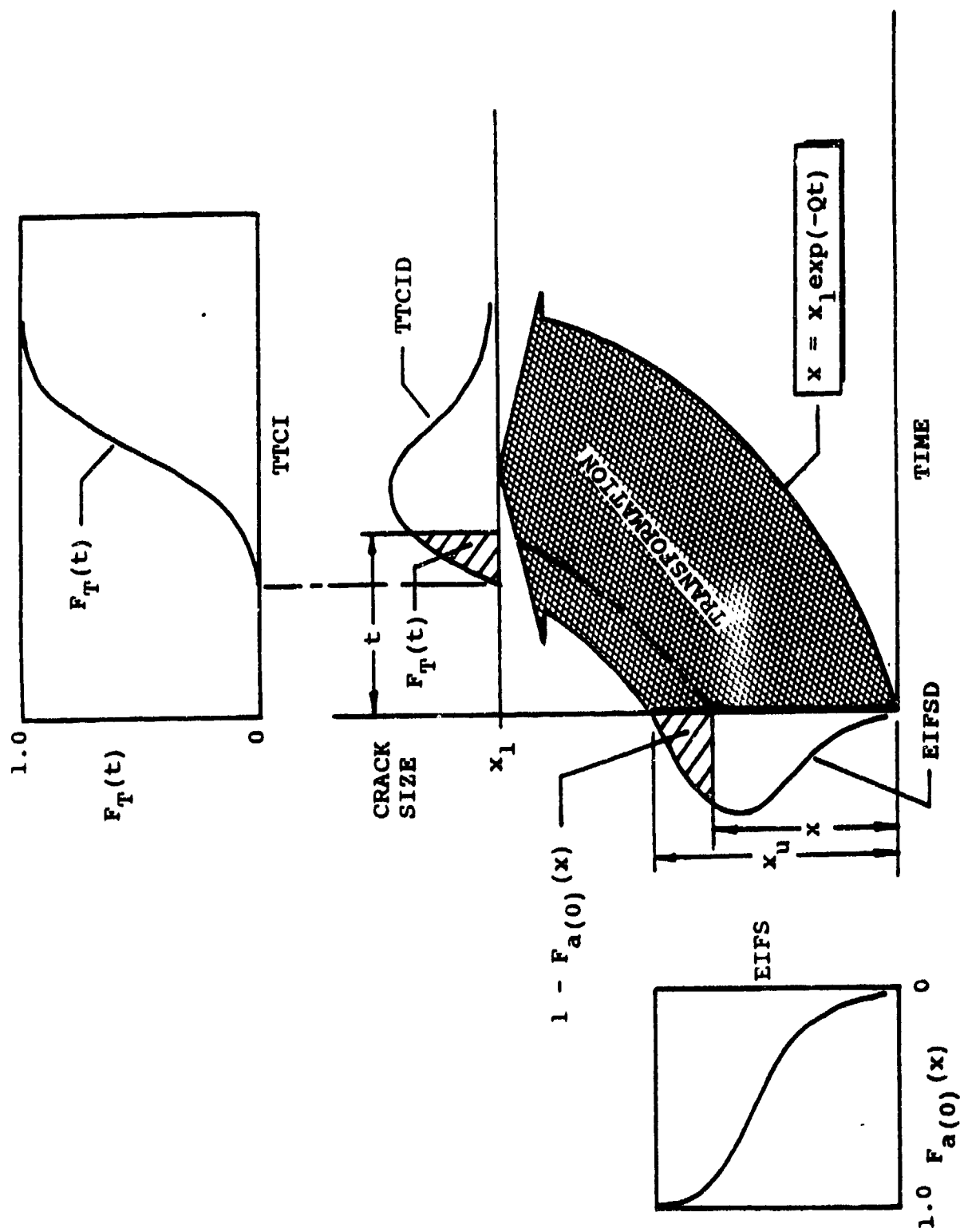


Figure 3. Transformation of the Cumulative Distribution of EIFS,  $F_a(0)(x)$ , to Obtain the Cumulative Distribution of TTCl,  $F_T(t)$ .

the following characteristics: (1) an EIFS is an artificial crack assumed to represent the initial fatigue quality of a structural detail in the as-manufactured condition whatever the source of fatigue cracking may be, (2) it has no direct relationship to actual initial flaws in fastener holes such as scratches, burrs, microdefects, etc., and it cannot be verified by NDI, (3) it has a universal crack shape in which the crack size is measured in the direction of crack propagation, (4) EIFSs are in a fracture mechanics format but they are not based on linear elastic fracture mechanics (LEFM) principles and are not subject to such laws or limitations such as the "short crack effect," (5) it depends on the fractographic data used, the fractographic crack size range used for the back-extrapolation and the crack growth rate model used, (6) it must be grown forward in a manner consistent with the basis for the EIFS, (7) EIFSs are not unique - a different set is obtained for each crack growth law used for the back-extrapolation, and (8) EIFSs are not necessarily comparable with minimum crack sizes obtained from fractographic results from tear-down inspections.

An EIFSD is an "engineering approach" for representing IFQ. It is not based on mechanistic theories for microstructural behavior. This approach for representing the IFQ, when properly used, can provide reasonable durability analysis results for metallic airframes.

Since EIFSs are determined by back-extrapolating fractographic results, they are not strictly "generic," because the fractographic results are obtained from fatigue cracks generated under specified test conditions (e.g., load spectrum, stress level, % bolt load transfer, fastener hole type/fit, etc.). Therefore, the EIFSs depend on the test conditions reflected. The real issue of the EIFS approach is not whether the EIFSs or EIFSDs are "generic" or not but rather can reasonable durability analysis predictions for  $p(i,T)$  and/or

$F_T(t)$  be obtained for a given set of conditions using EIFSs based on another set of conditions (e.g., different load spectrum, stress level, % bolt load transfer, etc.)? Under this program we have shown that fractographic data pooling and statistical scaling techniques can be effectively used to develop meaningful EIFSDs for practical durability analyses. Hence, the EIFSD can be established for a given set of conditions and the same EIFSD can be used to make reasonable predictions for another set of conditions. Data pooling and statistical scaling techniques are described and discussed in Sections 2.3.1 and 2.3.2, respectively.

Durability analysis concepts, issues and philosophy are discussed in detail in Appendix A. This includes the current understanding of EIFS concepts, principles and practices.

### 2.3 ESTIMATION OF EIFSD PARAMETERS

The following Weibull compatible EIFSD parameters in Eq. 1 must be estimated and optimized to define IFQ:  $x_u$ ,  $\alpha$  and  $\phi$ . In this section methods are presented for estimating these parameters, including procedures for pooling fractographic results, statistical scaling, optimization and checking suitability of the candidate EIFSD for durability analysis.

The methodology for determining a suitable EIFSD for durability analysis includes several elements: (1) selection and use of fractographic data, (2) fractographic crack size range, AL-AU, and reference crack size,  $x_1$  for time-to-crack-initiation (TTCI), (3) deterministic crack growth rate model, (4) crack size-time relationship or "EIFS Master Curve", (5) EIFSD function (e.g., Weibull compatible or lognormal compatible), (6) determination of EIFSs and/or TTCIs, (7) statistical scaling procedure for obtaining homogenous data, (8) estimation of EIFSD parameters based on data pooling procedure (i.e., combined least square sums approach (CLSSA) or

method of moments approach), (9) iterative procedure for optimizing the EIFSD parameters and (10) justification of candidate EIFSD for durability analysis.

Detail procedures and methods for estimating and optimizing the EIFSD parameter are given in the following subsections and in Appendices B, C, E, F and G. The general procedure for defining IFQ is summarized below and key elements are described in Fig. 4.

1. Select suitable EIFSD function for representing the initial fatigue quality (e.g., Weibull compatible or Lognormal compatible).

2. Select fractographic data set(s) to be used to determine the EIFSD. The data sets should be for the same material, same type load spectrum (e.g., fighter, bomber or transport) and type fastener/ hole/fit (i.e., straight bore or countersunk).

3. Select fractographic crack size range, AL-AU, to be used to determine the crack size-time relationship from a suitable deterministic crack growth rate model. Also select a reference crack size,  $x_1$ , for the TTCI's for each fractographic data set.

4. Estimate the crack growth parameter,  $Q_1$ , or "pooled Q", value for each fractographic data set using the fractographic data in the AL-AU crack size range (see Appendix E). A DCGA-based crack size-time relationship or EIFS master curve is defined in terms of  $Q_1$ . This relationship defines the equivalent initial flaw size (EIFS) corresponding to a given crack size  $x_1$  at time T or vice versa.

5. Compute TTCI's for each data set(s) using the reference crack size,  $x_1$ .



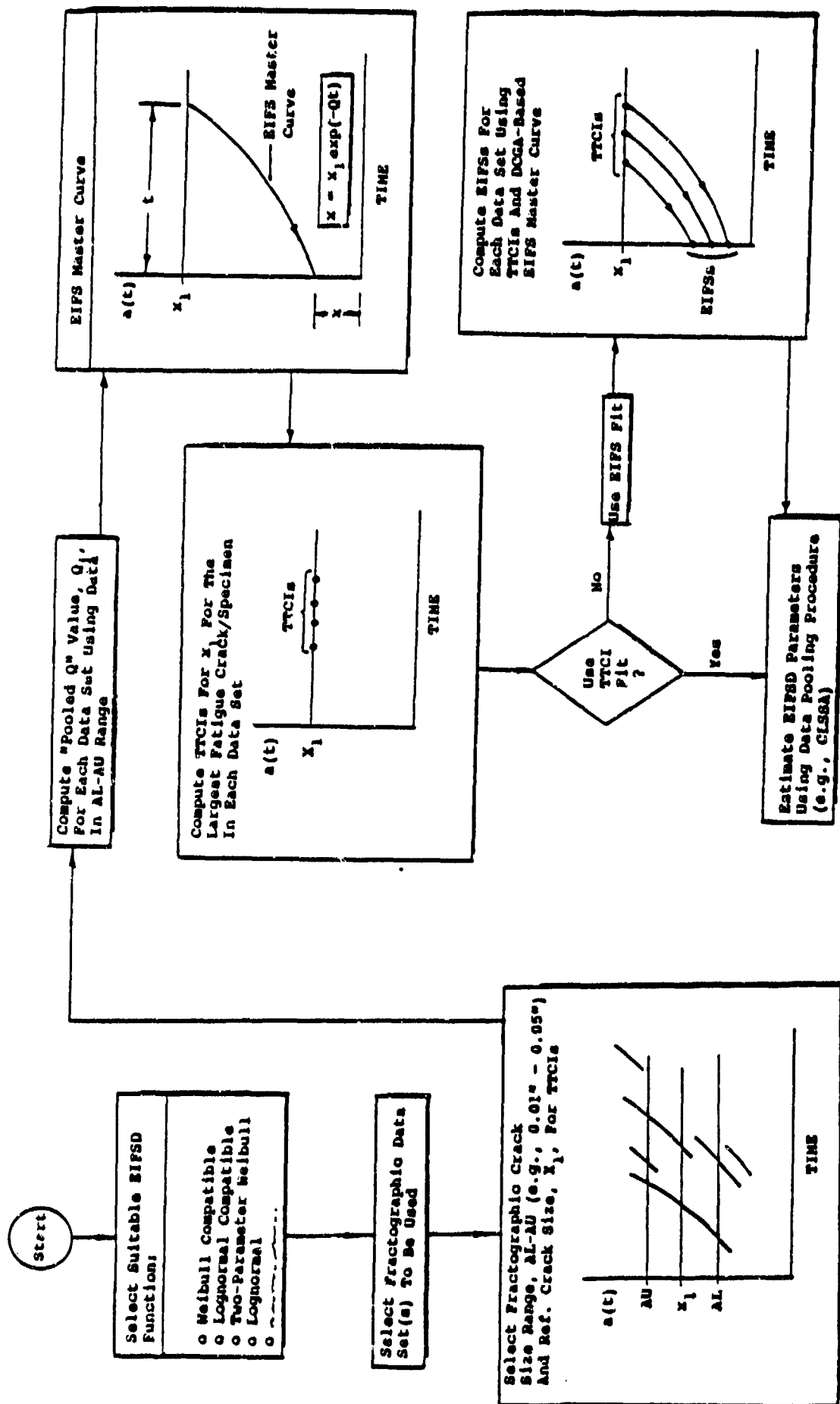


Figure 4. Initial Steps in Procedure Leading to Estimation of EIFSD Parameters.

6. Select either the "TTCI fit" or the "EIFS fit" approach to fit the EIFSD parameters. Refer to Appendix F.

7. If the EIFSs are to be used to fit the EIFSD parameters, compute the EIFSs for each fractographic data set using the TTCIs from step 5 and the EIFS master curve from step 4. Each EIFS in a given data set is determined using the same crack size-time relationship or EIFS master curve. Hence, each EIFS in the data set has a corresponding TTCI and vice versa.

8. Select the data pooling-parameter fitting method to be used to estimate the EIFSD parameters: (1) combined least square sums approach (CLSSA) or (2) method of moments approach. Both approaches include a scaling procedure for making the fractographic results homogeneous for different data sets. With scaling, results for different fractographic data sets can be "mixed and matched" to estimate the EIFSD parameters in a global sense.

9. Optimize the EIFSD parameters using an iterative procedure. Details are given in Section 2.3.4.

10. Justify using the EIFSD for durability analysis. Correlate predictions for the cumulative distribution of TTCI and/or the cumulative distribution of crack size with fractographic results for selected data sets. The goodness-of-fit, based on the proposed EIFSD, should be checked using, as a minimum, data sets used to estimate the EIFSD. We recommend that other fractographic data sets also be used to check the "goodness-of-fit."

Computer software is available in Volume V [6] to estimate, optimize and justify the EIFSD parameters for durability analysis, including a goodness-of-fit plotting capability.

### 2.3.1 Data Pooling

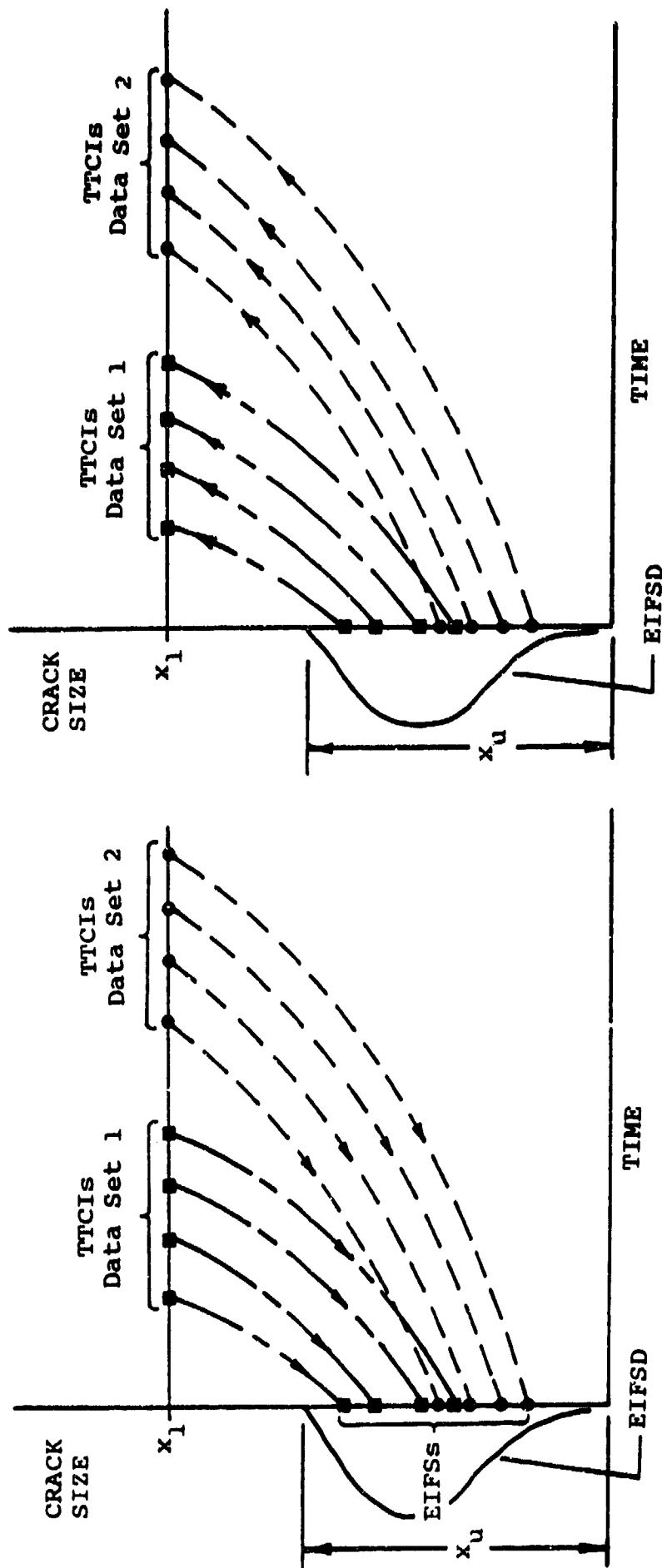
Data pooling is a practical way to define the EIFSD parameters for different data sets (e.g., same type of load spectrum but different stress levels and  $\frac{1}{2}$  bolt load transfer). Pooling effectively increases the sample size and confidence in the EIFSD parameters. Likewise, this is a reasonable approach for justifying an EIFSD for more general applications. This is an important perspective for practical durability analysis.

The fractographic data pooling concept is conceptually illustrated in Fig. 5 using two fractographic data sets. The applicable TCI values for each data set are represented by a TTCID as shown in Fig. 5. The basic idea of data pooling is that the crack growth results for each data set have a common EIFSD. For example, if the TCIs for each data set are regressed backwards to time zero using the applicable EIFS master curve for each data set, the resulting EIFSs have the same EIFSD. Here's another way to look at this. The TTCID for different fractographic data sets can be determined using the same EIFSD.

The EIFSD parameters can be estimated using either an "EIFS fit" or a "TCI fit." Either approach will give the same EIFSD parameters. There are subtle differences between the two approaches.

With the "EIFS fit," TCIs for a given reference crack size,  $x_1$ , are back-extrapolated to time zero using the applicable EIFS master curve (e.g., Eq. 4) to obtain the EIFSs for each data set. The resulting EIFSs and a "fitting procedure" are then used to estimate the EIFSD parameters in a global sense for the particular EIFSD function used (see Fig. 6(a)), i.e., Eq. 1. Different "fitting procedures" have been inves-





(a) Estimate EIFSD Parameters Using EIFSS

(b) Estimate EIFSD Parameters Using TTCIS

Figure 6. Two Different Philosophies for Estimating EIFSD Parameters Using Fractographic Data Pooling Procedures and DCGA.

tigated but only the combined least square sums approach will be considered in Section II (ref. Section 2.3.3). Other "fitting procedures" are described and discussed in Appendix F.

A general expression for the cumulative distribution of TTCI,  $F_T(t)$ , is given in Eq. 7. This expression is based on a transformation of the Weibull compatible EIFSD using the EIFS master curve given in Eq. 4 (see Fig. 3). Equation 7 contains all the EIFSD parameters in Eq. 1 (i.e.,  $x_u$ ,  $\alpha$  and  $\phi$ ) plus the crack growth rate parameter  $Q$ . Therefore, with the "TTCI fit" the EIFSD parameters can be estimated using TTCI values for each data set considered in the data pooling procedure (Fig. 6(b)).  $Q$  can be obtained from fractographic results and/or using a suitable analytical crack growth program [e.g., 38, 39]. Detailed procedures for computing  $Q$  are given in Appendix E.

With the "TTCI fit" the EIFSD parameters can be estimated without computing EIFSs. Since TTCIs are actual physical quantities and EIFSs are not, some may prefer to quantify the EIFSD parameters using TTCI values. The same  $Q$  value is used for both the "EIFS fit" and the "TTCI fit." In any case, it can be shown that the same EIFSD parameters are obtained from a given  $x_u$  value using either approach.

### 2.3.2 Statistical Scaling

The IFQ or EIFSD for fastener holes is defined for an "equivalent single hole population." Therefore, the fatigue cracking resistance of each fastener hole in each test specimen is accounted for in the definition of the EIFSD. Test specimens for acquiring fatigue crack growth data may have one or more fastener holes per specimen. Some specimens may or may not be fatigue tested to failure. Also, every fastener hole in each replicate test specimen may not contain a

measurable fatigue crack or else the crack is too small or complex (e.g., multiple crack origins and branching) for fractographic analysis. A statistical scaling technique has been developed (see Appendix B for details) for determining the EIFSD for an "equivalent single hole population" based on the largest fatigue crack per specimen. Hence, it is necessary to read only the fractographic results for the largest crack per specimen. Essential elements are conceptually described in Fig. B.1. This technique is very general and is independent of the distribution functions used. It accounts for the number of fastener holes per test specimen in a given fractographic data set. It minimizes the fractographic reading requirements, permits a maximum utilization of the available fractographic data and allows for "mixing and matching" of fractographic data for the largest crack in specimens with a different number of holes.

Details of the statistical scaling technique developed are given in Appendix B. Essential features and key equations are summarized in the following.

Let the cumulative distribution of EIFS for a single hole population be denoted by  $F_{a(0)}(x)$ , and that of the maximum EIFS based on the largest fatigue crack per specimen with  $\ell$  fastener holes, be denoted by  $F_{a_\ell(0)}(x)$ . Assuming that fatigue cracking in each fastener hole of a specimen is statistically independent of the other holes,  $F_{a_\ell(0)}(x)$  is related to  $F_{a(0)}(x)$  through the following,

$$F_{a_\ell(0)}(x) = \left[ F_{a(0)}(x) \right]^\ell \quad (8)$$

where  $\ell$  = number of fastener holes per specimen. Similar expression for the cumulative distribution of TCI are given in Eqs. 9 and 10.

$$F_T(t) = 1 - \left[ 1 - F_{T_\ell}(t) \right]^{1/\ell} \quad (9)$$

$$F_{T_\ell}(t) = 1 - \left[ 1 - F_T(t) \right]^\ell \quad (10)$$

where  $F_T(t)$  = cumulative distribution of TTCI for an equivalent single hole population, and  $F_{T_\ell}(t)$  = cumulative distribution of the minimum TTCI in  $\ell$  holes.

In a similar manner, let  $F_a(t)(x)$  denote the cumulative distribution of crack size at any time  $t$  for an "equivalent single hole population" for  $F_{a(0)}(x)$ . Let  $F_{a_\ell}(t)(x)$  denote the cumulative distribution of crack size at any  $t$  based on the EIFS cumulative distribution,  $F_{a_\ell(0)}(x)$ . Then,  $F_a(t)(x)$  is related to  $F_{a_\ell}(t)(x)$  as follows.

$$F_{a_\ell}(t)(x) = \left[ F_a(t)(x) \right]^\ell \quad (11)$$

Let  $F_T(t)$  denote the cumulative distribution of TTCI for an "equivalent single hole population" and  $F_{T_\ell}(t)$  denote the cumulative distribution of TTCI based on the smallest TTCI per specimen with  $\ell$  fastener holes. Then, the following relationship between  $F_T(t)$  and  $F_{T_\ell}(t)$  holds.

$$F_{T_\ell}(t) = 1 - \left[ 1 - F_T(t) \right]^\ell \quad (12)$$

The simple scaling technique described in this section has been incorporated into the procedure for estimating the EIFSD parameters for one or more fractographic data sets. Computer software for the IBM compatible PC is available for estimating the EIFSD parameters, for checking goodness-of-fit and for plotting results. The statistical scaling technique is evaluated in Volume II [3].



### 2.3.3 Combined Least Square Sums Approach (CLSSA)

The data pooling and statistical scaling concepts described in Sections 2.3.1 and 2.3.2, respectively, are important considerations for estimating the EIFSD parameters for fastener holes or other types of structural details. Data pooling is important for two reasons: (1) the sample size is increased and so is the confidence in the estimated EIFSD parameters and (2) the derived EIFSD can be justified for a broader range of variables (e.g., stress level, % bolt load transfer, etc.) than that for a single fractographic data set.

The desired EIFSD should account for the fatigue cracking resistance of each fastener hole in each test specimen for each fractographic data set to obtain a "single hole population basis." Test specimens for one fractographic data set may have a different number of fastener holes than another data set. A method is needed for "mixing and matching" or pooling the fractographic results for different data sets so that the EIFSD parameter (e.g.,  $x_u$ ,  $\alpha$  and  $\phi$  in Eq. 1) can be estimated in a global sense. A statistical scaling technique is described in Section 2.3.2 and Appendix B for scaling the fractographic results of test specimens with one or more fastener holes to an "equivalent single hole population" basis.

In this section, a method, referred to as the "combined least square sums approach" (CLSSA), is presented for estimating EIFSD parameters for one or more fractographic data sets. This data pooling method accounts for statistical scaling. In this section the CLSSA is described and discussed for the Weibull compatible EIFSD function. Other EIFSD functions and methods are described in Appendix F.

The EIFSD parameters for the Weibull compatible EIFSD function (i.e.,  $x_u$ ,  $\alpha$  and  $\phi$ ) can be estimated using either TTCI or EIFS input data for one or more fractographic data sets. Either approach will give the same EIFSD parameter values. The TTCIs or EIFSs (whichever is used) for each fractographic data set are ranked and least square sums are obtained for each data set using the applicable EIFSD function and data used to fit the EIFSD parameters. Each least square sum accounts for the scaling factor for each data set. The combined least square sums for all data sets are obtained by summing the applicable value for each data set. Then, the EIFSD parameters are estimated using the combined least square sums and a global fit.

The procedure for implementing the CLSSA is given below:

1. Select fractographic data sets to be used in the pooling procedure to estimate the EIFSD parameters in a global sense.
2. Acquire the TTCIs or EIFSs for each data set to be used to estimate the EIFSD parameters.
3. Define scaling factor,  $\ell_x$ , for each data set to be used.
4. Select the EIFSD function to be used and then transform it into a linear least square fit form. If EIFSs are used for the fit, use the expression for the cumulative distribution of EIFS for the fit. On the other hand, if TTCIs are used for the fit, use the expression for the cumulative distribution of TTCI based on the transformation of the EIFSD and the crack growth model used (e.g., Eq. 2).
5. Assume a value for the EIFS upper bound within the allowable limits for fastener holes: largest EIFS in any

data set  $\leq x_u \leq 0.05$ ". Compute the combined least square sums for all data sets combined. Use these results to estimate the EIFSD parameters.

6. The EIFSD parameters can be optimized within the  $x_u$  constraints. A reasonable way to do this is to minimize the total standard error for all data sets used to determine the EIFSD parameters. The optimum  $x_u$  can be determined by trial and error until the total standard error has been minimized.

The CLSSA is described in detail in Section F.2. Essential elements and equations, based on the Weibull compatible EIFSD, are summarized in the following for both the "TTCI fit" and "EIFS fit" approaches.

It has been demonstrated in Volume II [3] that the "TTCI fit" and the "EIFS fit" will give the same EIFSD parameter values. From a philosophical standpoint, the TTCI fit is recommended because TTCIs can be directly verified from the fractographic data, and EIFSs do not have to be computed; whereas, the EIFSs are artificial quantities that cannot be verified by NDI. If the EIFS fit is used the resulting EIFSD has to be grown forward anyway to test the goodness-of-fit of the  $F_a(t)(x)$  and  $F_T(t)$  predictions. It's all a matter of choice - either method is acceptable. Software is available in Volume V [6] for an IBM compatible PC to implement either method for the Weibull compatible EIFSD function.

#### 2.3.3.1 TTCI Fit.

The time-to-crack initiation (TTCI) is defined as the time required to initiate a specified crack size,  $x_1$ , in a structural detail in the as-manufactured condition. For example, in Fig. 7 fractographic results are depicted for three fatigue cracks. The time to initiate a crack size,  $x_1$ , for each of these tracks is  $t_1$ ,  $t_2$  and  $t_3$ , respectively, as shown in Fig. 7.

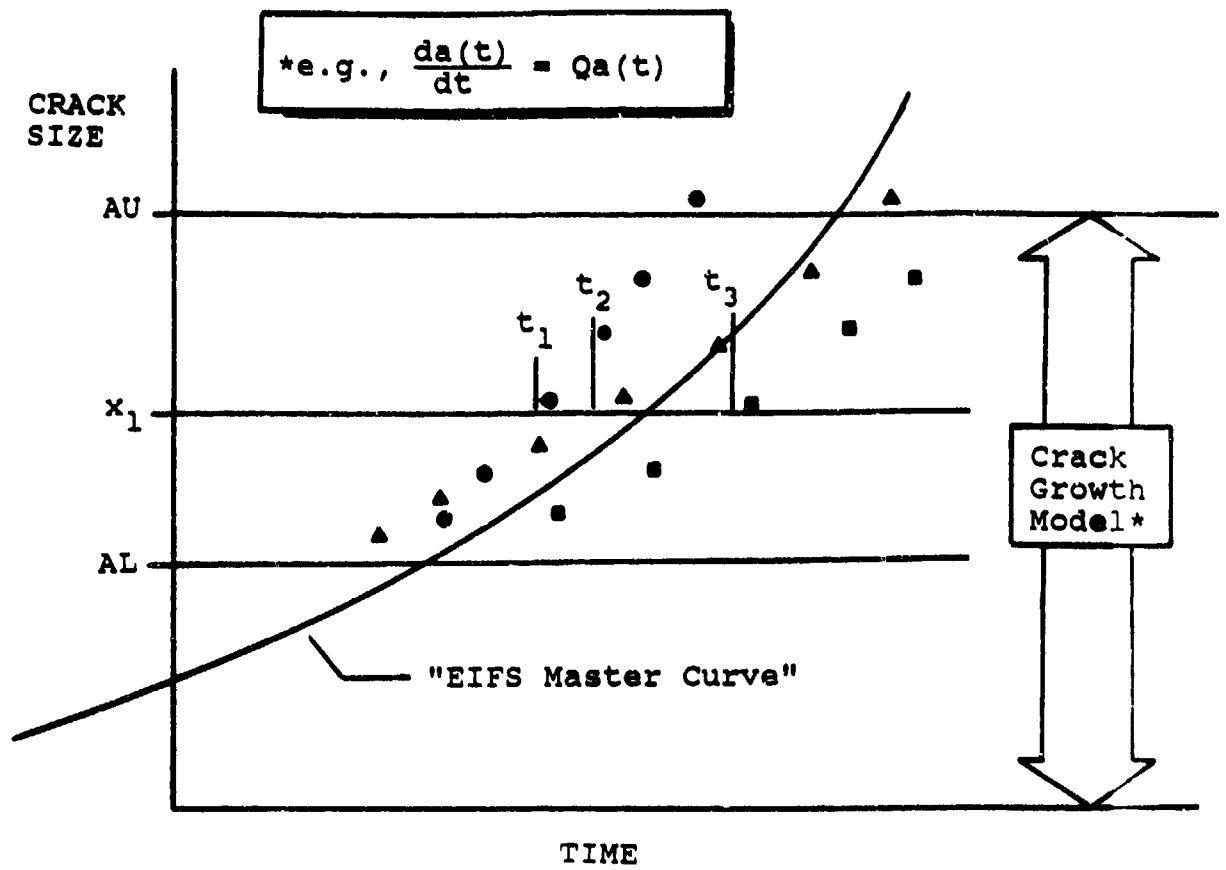


Figure 7. Time-To-Crack-Initiation (TTCI) Concept.

TTCIs can be determined from the fractographic results in one of the following ways: (1) directly from the fractographic results when the measured crack size is exactly equal to  $x_1$ , (2) by interpolation (e.g., three-point Lagrangian), and (3) by extrapolation (see Section G.3 for details).

The Weibull compatible EIFSD parameters  $\alpha$  and  $\phi$  can be determined for a given  $x_u$  using Eq. 13 and 14, respectively.

$$\alpha = \frac{\sum_{i=1}^M N_i \sum_{j=1}^M \sum_{l=1}^{N_i} X_{ij} Y_{ij} - \sum_{i=1}^M \sum_{j=1}^{N_i} X_{ij} \sum_{l=1}^M \sum_{k=1}^{N_l} Y_{lk}}{\sum_{i=1}^M N_i \sum_{j=1}^M \sum_{l=1}^{N_i} X_{ij}^2 - \left[ \sum_{i=1}^M \sum_{j=1}^{N_i} X_{ij} \right]^2} \quad (13)$$

$$\phi = \exp \left\{ \frac{\alpha \sum_{i=1}^M \sum_{j=1}^{N_i} X_{ij} - \sum_{i=1}^M \sum_{j=1}^{N_i} Y_{ij}}{\alpha \sum_{i=1}^M N_i} \right\} \quad (14)$$

The terms in Eqs. 13 and 14 are defined as shown in Eq. 15. The expression for the total standard error is given in Eq. 16.

$$\left. \begin{aligned} X_{ij} &= \ln \left[ \ln (x_u/x_1) + Q_i t_{ij} \right] \\ Y_{ij} &= \ln \left\{ - (1/l_i) \ln \left[ 1 - F_{T_i}(t_{ij}) \right] \right\} \\ B &= -\alpha \ln \phi \\ F_{T_i}(t_{ij}) &= j/(N_i + 1); j = 1, 2, \dots, N_i \end{aligned} \right\} \quad (15)$$

$$TSE = \sqrt{\frac{\sum_{i=1}^M \sum_{j=1}^{N_i} \left\{ \frac{j}{(N_i + 1)} - 1 + \exp \left\{ -l_i \left[ \frac{\ln(x_u/x_1) + Q_i t_{ij}}{\phi} \right] \alpha \right\} \right\}^2}{\sum_{i=1}^M N_i}} \quad (16)$$

In Eqs. 13 through 16,  $\ell_i$  = number of fastener holes for the replicate specimen in data set  $i$ ,  $M$  = number of fractographic data sets used in the pooling procedure,  $N_i$  = number of TTCI values for the  $i$ th data sets,  $x_u$  = EIFS upper bound limit,  $x_l = a_0$  = reference crack size for TTCIs,  $Q_i$  = pooled  $Q$  value for the  $i$ th data set (see Eq. 2),  $t_{ij}$  =  $j$ th TTCI value for the  $i$ th data set,  $F_{T_{\ell_i}}(t_{ij})$  =  $j$ th value for the cumulative distribution of TTCI minimums (i.e., based on the largest fatigue crack per specimen in each data set) for the  $i$ th data set. Refer to Appendix F for details.

The Weibull compatible parameters  $\alpha$  and  $\phi$  for a given  $x_u$  can be determined using the same equations used for the TTCI fit approach (i.e., Eqs. 13 and 14). For the EIFS fit the expressions shown in Eq. 17 should be used in Eq. 13 and 14.

$$\left. \begin{aligned} X_{ij} &= \ln \ln (x_u / x_{ij}) \\ Y_{ij} &= \ln \left[ - (1/\ell_i) \ln F_{a_{\ell_i}(0)}(x_{ij}) \right] \\ B &= -\alpha \ln \phi \\ F_{a_{\ell_i}(0)}(x_{ij}) &= j/(N_i + 1); j = 1, 2, \dots, N_i \end{aligned} \right\} \quad (17)$$

The expression for the total standard error, TSE, is given in Eq. 18.

$$TSE = \sqrt{\frac{\sum_{i=1}^M \sum_{j=1}^{N_i} \left\{ \frac{j}{(N_i + 1)} - \exp \left[ -\ell_i \left| \frac{\ln (x_u / x_{ij})}{\phi} \right| \alpha \right] \right\}^2}{\sum_{i=1}^M N_i}} \quad (18)$$

In Eqs. 17 and 18 all terms are defined the same way as those for Eq. 15 except the following:  $N_i$  = number of EIFS values for the  $i$ th data set,  $x_{ij}$  =  $j$ th EIFS value for the  $i$ th data set, and  $F_{a_{\ell_i}(0)}(x_{ij})$  =  $j$ th value for the cumulative distribution of EIFS maximums for the  $i$ th data set.

2.3.3.2 EIFS Fit. Methods for determining EIFSs using the DCGA are described in Appendix E. The recommended procedures for computing DCGA-based EIFSs are summarized below:

1. Select the fractographic data to be used.
2. Select the fractographic crack size range, AL-AU, to be used to compute the "pooled Q" value (e.g., 0.01"-0.05" is considered reasonable for defining IFQ for fastener holes).
3. Select a reference crack size,  $x_1$ , within the following constraint:  $AL \leq x_1 \leq AU$ . Then, determine the TTCIs for the largest fatigue crack for each specimen in the data set.
4. Compute the "pooled Q" value for the given fractographic data set using Eq. C-13. Q is based on fractographic results for the selected AL-AU range. The pooled Q value is used in Eq. 4 to define the crack size-time relationship for the data set. This relationship is used as an "EIFS master curve."
5. Use Eq. 4 to determine the EIFS corresponding to each TTCI in the data set as conceptually described in Fig. 8.

The EIFSs for one or more data sets can be used to fit the EIFSD parameters for the Weibull compatible EIFSD function. Methods and equations are given in detail in Appendix F.

#### 2.3.4 Optimization of Parameters and Goodness-of-Fit

The purpose of this section is to: (1) describe an iterative procedure for optimizing the EIFSD parameters for structural details and (2) describe a way to check the goodness-of-fit of the candidate EIFSD and justify it for durability analysis.

Note: Determine EIFSs corresponding to TTCIs for given reference crack size. Fit the EIFSD parameters using EIFSs.

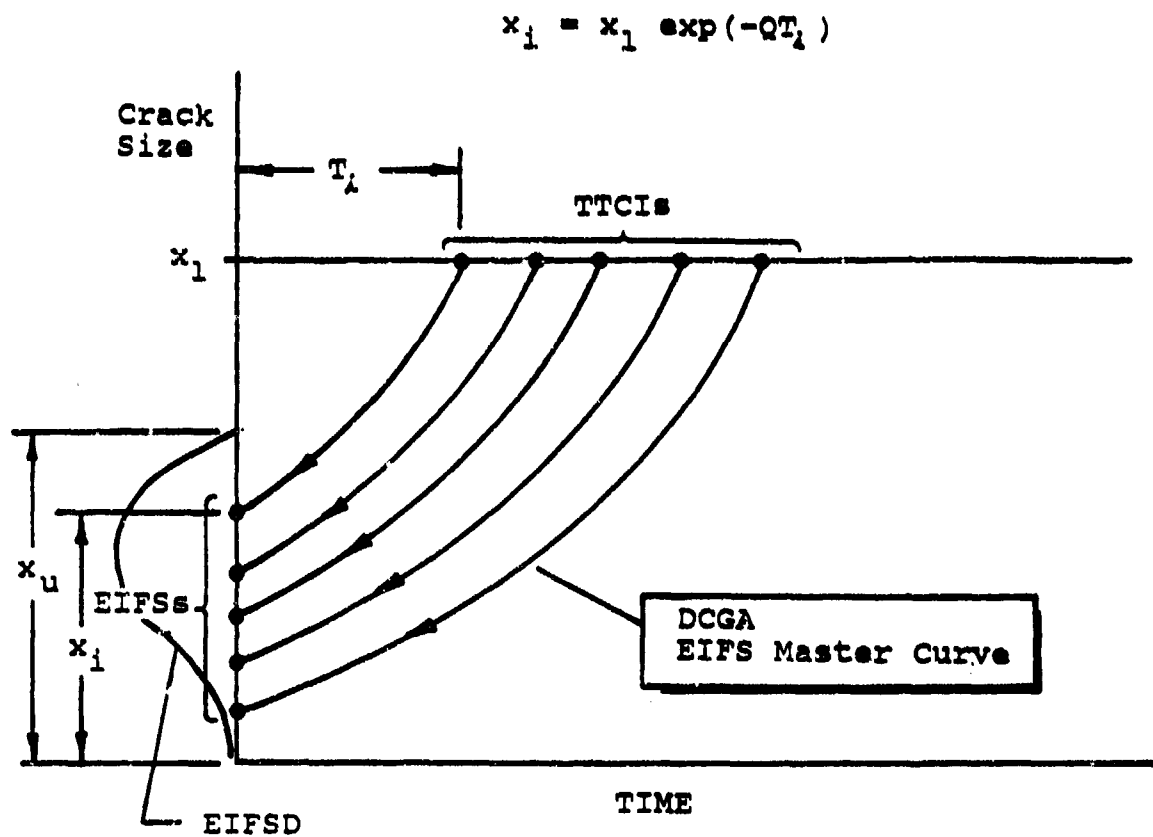


Figure 8. EIFS Approach for Estimating EIFSD Parameters.



A general procedure for optimizing the EIFSD parameters and checking goodness-of-fit for compatible type EIFSD functions is shown in Fig. 9. The following iterative approach is considered reasonable for optimizing the EIFSD parameters. An upper bound EIFS limit,  $x_u$ , is assumed with the following limits for fastener holes: largest EIFS in any data set  $\leq x_u \leq 0.05$ ". If the "TTCI fit" is used, the smallest TTCI for each data set can be converted to an EIFS using the EIFS-time relationship of Eq. 4. The parameters  $\alpha$  and  $\phi$  in the Weibull compatible EIFSD function, Eq. 1, can be estimated using the CLSSA described in Section 2.3.3. Then, the total sum squared error can be computed using either the TTCIs or EIFSs for those data sets used to estimate  $\alpha$  and  $\phi$ . The optimum  $\alpha$  and  $\phi$  for a given  $x_u$  is obtained when the total standard error, TSE, for all data sets used in the pooling procedure has been minimized within the allowable limits for  $x_u$  (i.e., largest EIFS in any data set  $\leq x_u \leq 0.05$ " for fastener holes). An iterative procedure is used to optimize  $x_u$ ,  $\alpha$  and  $\phi$ . Software is available in Volume V [6] to efficiently optimize the EIFSD parameters using the procedure described. After estimating the EIFSD parameters, the EIFSD needs to be tested to determine if it is suitable for representing the initial fatigue quality of the structural details in question.

The procedure used to estimate the EIFSD parameters accounts for the data fit. For example, suppose the CLSSA and a "TTCI fit" is used to estimate the EIFSD parameters. In this case, the EIFSD parameters are optimized in a least square sense using the cumulative distribution of TTCI. Further testing of the resulting EIFSD is recommended, including goodness-of-fit plots.

The EIFSD is determined using the fractographic results for one or more data sets. Each data set is assumed to have the same type spectrum (e.g., fighter, bomber or transport), fastener/hole/fit and material. Stress level, % bolt load transfer, and number of fastener holes can vary for each

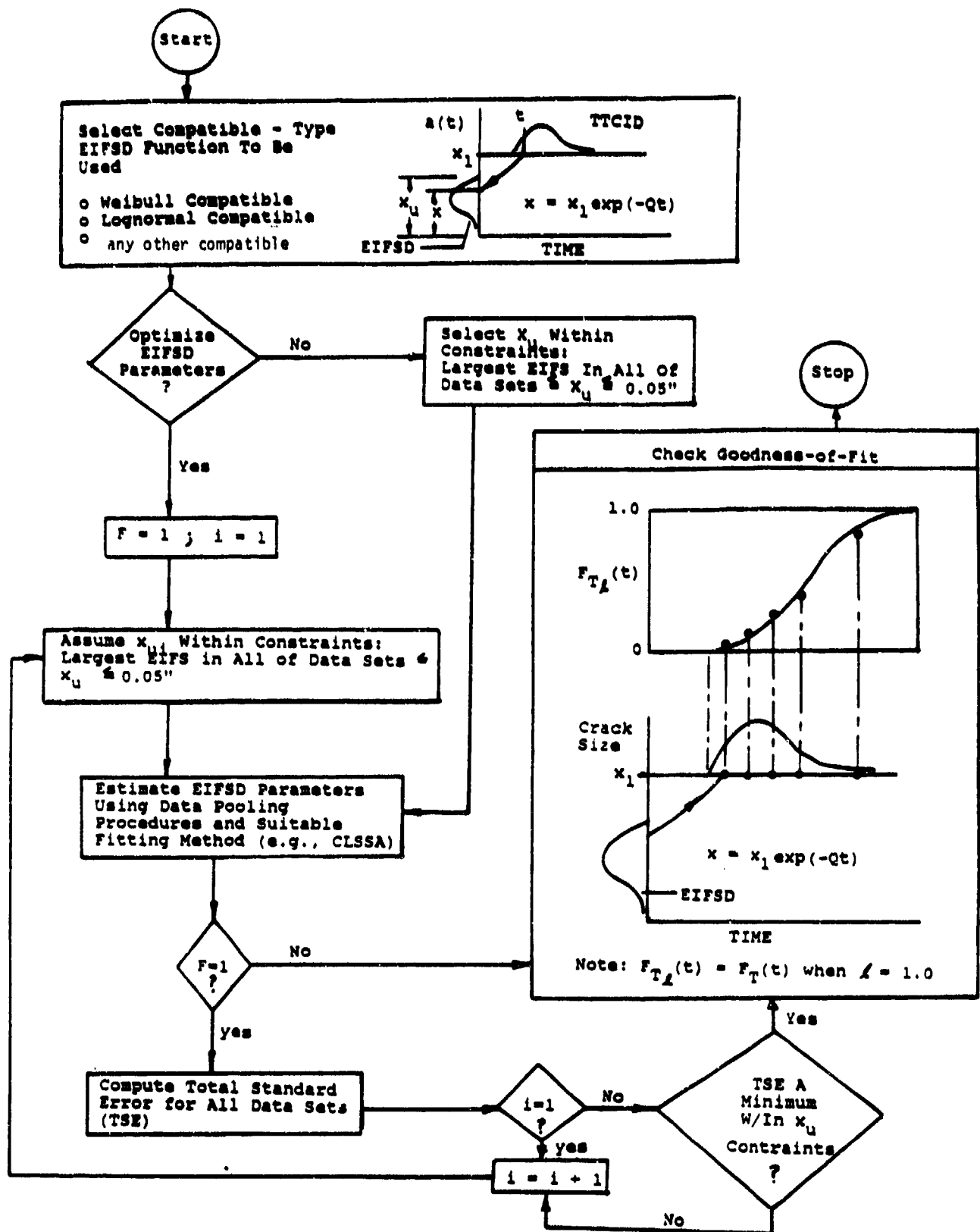


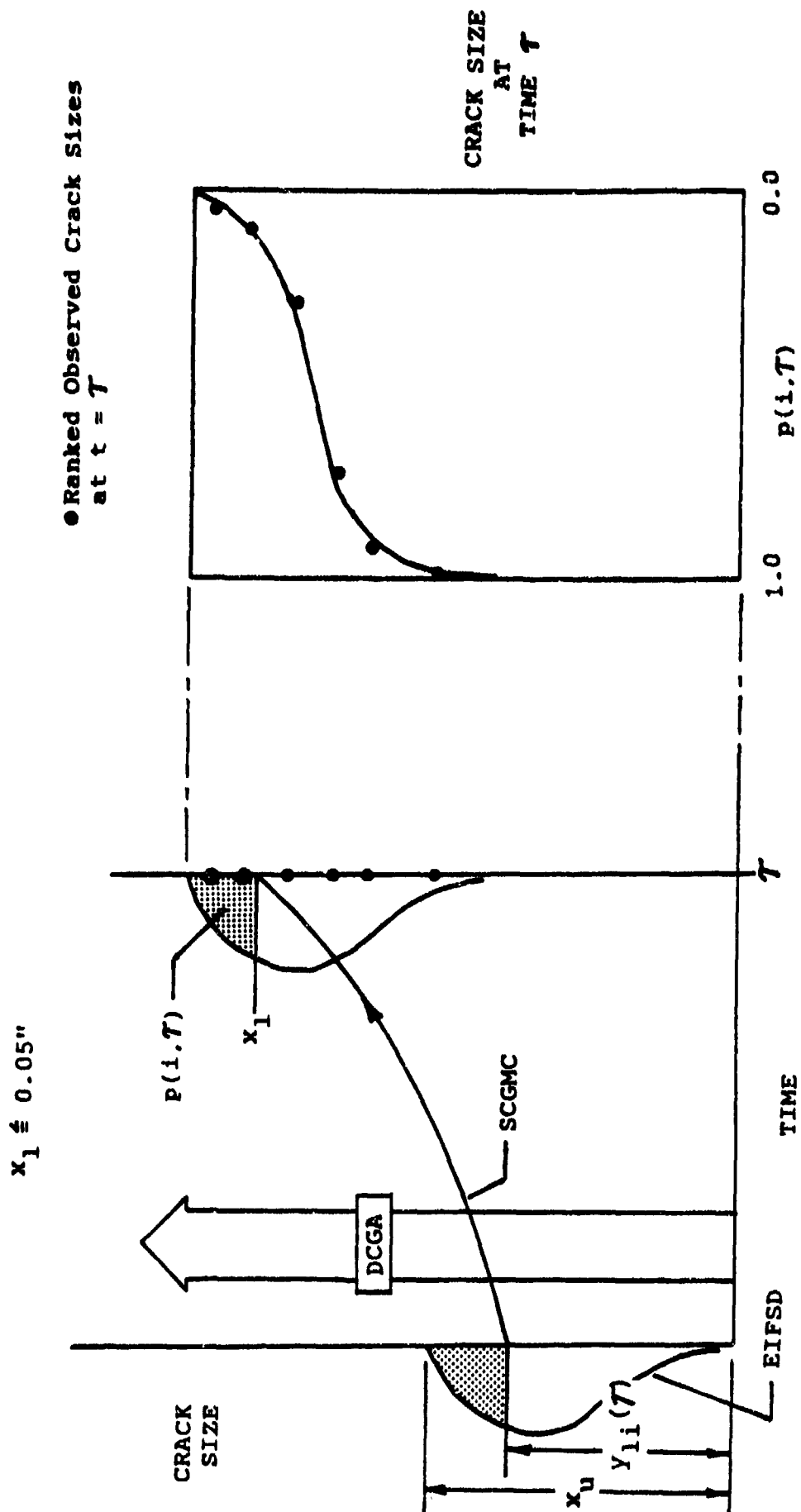
Figure 9. General Procedure for Optimizing EIFSD Parameters and Checking Goodness-of-Fit for Compatible Type EIFSD Function.

fractographic data set to be used to estimate the EIFSD parameters in a global sense. In most cases, the durability design conditions will not likely be the same as those reflected in any one of the fractographic data sets used to estimate the EIFSD parameters. The EIFSD is justified for other design conditions by showing that "reasonable" durability analysis results can be obtained for those fractographic data sets used to define the IFQ and for selected data sets not used to define the IFQ. In such case, it is assumed that the candidate EIFSD will be applied to the same or similar material, same type load spectrum and same type fastener/hole/fit. For durability analysis applications, stress level,  $\frac{1}{2}$  bolt load transfer and number of fastener holes can vary.

The EIFSD can be justified using the fractographic data used to estimate the EIFSD parameters and/or other fractographic data sets. For example, the "pooled Q" value for each fractographic data set to be considered and the candidate EIFSD can be used to predict the cumulative distribution of crack size at any time  $t$ ,  $F_a(t)(x)$ , or the cumulative distribution of TTCI at any crack size,  $x_1$ ,  $F_T(t)$ . The predicted cumulative distributions can be plotted and the results compared with the ranked fractographic observations for either crack size or TTCI. Such plots are conceptually depicted for the probability of crack exceedance and cumulative distribution of TTCI in Figs. 10 and 11, respectively.

The goodness-of-fit in the tail of the applicable cumulative distribution is of most interest. For example, the smaller TTCIs for a given crack size,  $x_1$ , and the largest crack sizes for a given time,  $t$ , are of most interest for judging goodness-of-fit.

Different criteria could be used to judge goodness-of-fit. For example, a visual inspection of the plotted results or the total standard error might be used.



(a) Probability of Crack Exceedance

(b)  $P(1, \tau)$  Goodness-of-Fit

Figure 10. Probability of Crack Exceedance and Goodness-of-Fit for Small Crack Size Range.

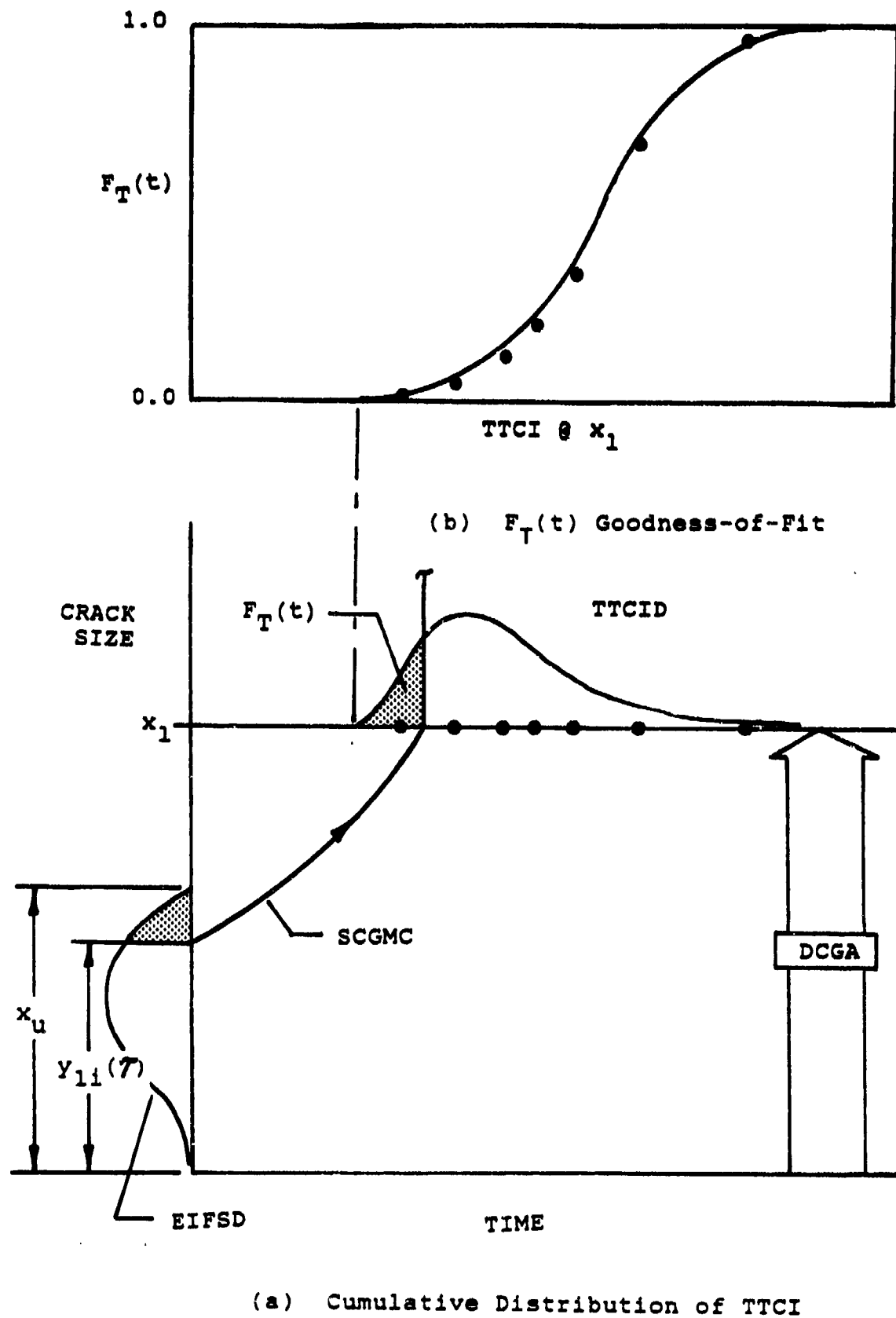


Figure 11. Cumulative Distribution of TTCI and Goodness-of-Fit for Small Crack Size Range.

## 2.4 DURABILITY ANALYSIS PREDICTIONS

Durability analysis predictions for the probability of crack exceedance,  $p(i, \tau)$ , and the cumulative distribution of TTCI,  $F_T(t)$ , can be determined using an appropriate EIFSD function,  $F_a(0)(x)$ , and a suitable service crack growth master curve (SCGMC). Guidelines and procedures are presented in this section for determining the SCGMC and for making durability analysis predictions for  $p(i, \tau)$  and  $F_T(t)$  in the small crack size range (e.g.,  $<0.10$ " ).

### 2.4.1 Service Crack Growth Master Curve

A Service Crack Growth Master Curve (SCGMC) describes the deterministic crack size-time relationship for a given stress region. It is used to grow the EIFSD forward to predict the following: (1) crack size at any time,  $t$ ; (2) the probability of exceeding a given crack size,  $x_1$ , at any time,  $t$ ; and (3) the cumulative distribution of TTCI at any reference crack size,  $a_0$ . The SCGMC depends on the conditions to be reflected in the durability analysis (e.g., loading spectrum, stress level, % bolt load transfer, etc.) and the basis for the EIFS or EIFSD to be used.

A crack initiation process takes place in the small crack size range. Eventually, as a dominant fatigue crack emerges, a crack propagation process takes over. The actual crack size where the process changes from initiation to propagation is debatable. Suppose we assume that crack sizes below 0.05" are governed by the initiation process and larger cracks are governed by the crack propagation process. In this context, the empirical crack size-time relationship for the small crack size range, referred to as the "service crack growth master curve" (SCGMC), then represents the initiation process. This is an "engineering approach" to a complex problem. Yet, such an approach is justified for durability analysis until such time that suitable mechanistic-based models are developed for practical applications.

In most durability design situations, a suitable LEFM analytical crack growth program [e.g., 38, 39] is used to develop the SCGMC for the desired analysis conditions because applicable fractographic results will not likely be available. The following general procedure for developing a SCGMC is recommended for durability analysis applications in the small crack size region.

1. Define the basis for the EIFSD to be used in the durability analysis. For example: (1) what fractographic crack size range, AL-AU, was used?; (2) basis for EIFS master curve (e.g., given empirical crack growth law, LEFM program applied without crack size restrictions or fracture mechanics-based program that included the "short crack effect"); and (3) what method was used to define the EIFS master curve, including criterion for goodness-of-fit and crack shape?

NOTE: An empirical EIFS-TTCI relationship (e.g., Eq. 4) is recommended for general applications to assure consistent EIFSDs will be obtained by different aerospace contractors for the same fractographic data base.

2. Use a suitable analytical crack growth program to "curve fit" or tune the EIFS master curve or curves (where the fractographic data pooling procedure is used) in the fractographic crack size range, AL-AU, used to define the EIFSD. In principle, it doesn't matter if the analytical crack growth program is limited to LEFM applications or not because crack sizes will be limited to sizes where LEFM principles apply anyway. The "curve fit" to the EIFS master curve in the selected AL-AU range is accomplished as follows: (1) plot the EIFS master curve to cover the applicable AL-AU range; (2) select a crack growth model (e.g., Walker- $\Delta K$ , Forman, etc.); (3) select  $da/dN$  versus  $\Delta K$  data and calibrate the crack growth model parameters for given material; (4) select

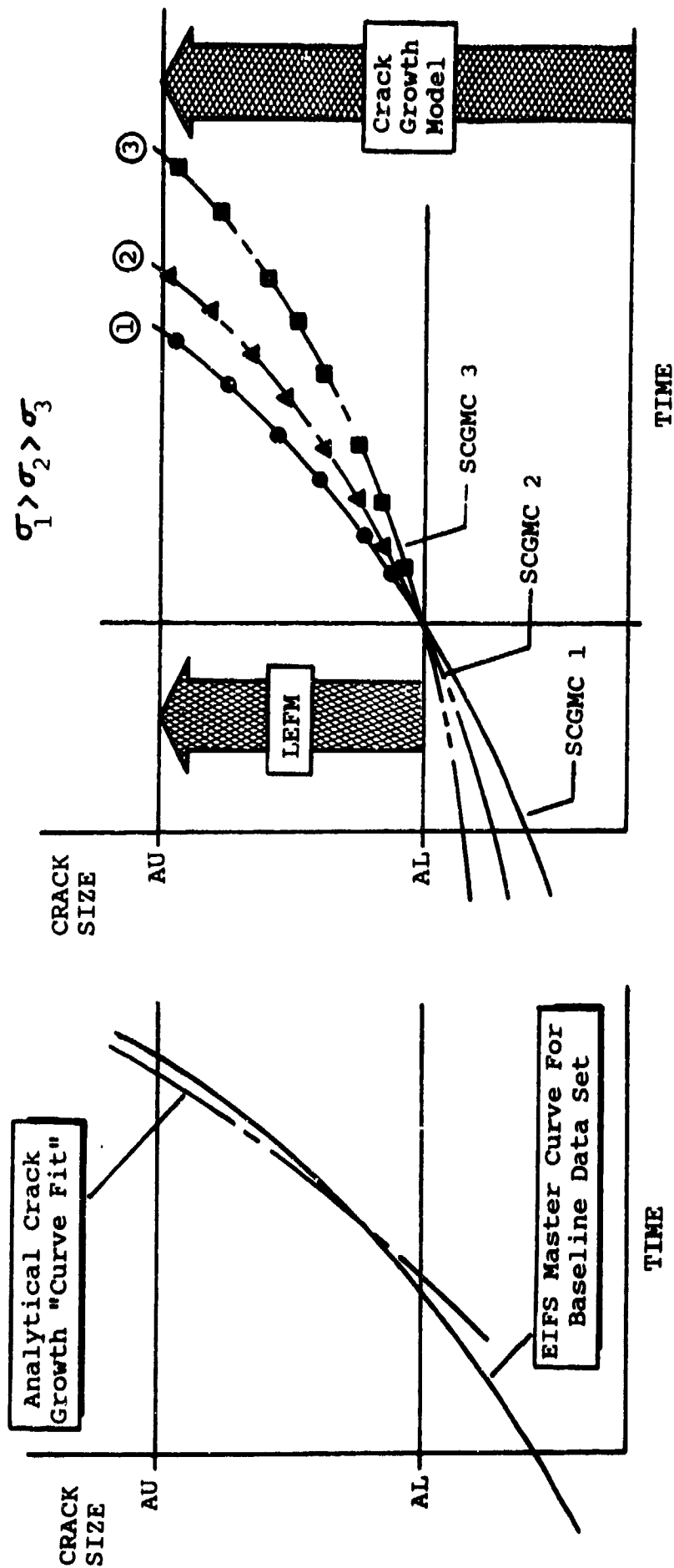
a load-retardation model (e.g., modified Willenborg, Wheeler, etc.); and (5) by trial and error, determine the remaining model parameters required to obtain a reasonable curve fit using the analytical crack growth program. The goodness-of-fit is determined subjectively. The above procedure is conceptually described in Fig. 12(a). The "curve fit" is accomplished using the applicable conditions reflected in the EIFS master curve (i.e., load spectrum, stress level, & bolt load transfer, hole type/diameter).

3. The tuned analytical crack growth program is then used to predict the crack growth over the applicable AL-AU range using the applicable durability analysis conditions. For example, a specific spectrum, stress level and & bolt load transfer and assumed crack shape/geometry are used to predict the crack size at a given time. This step is conceptually illustrated in Fig. 12b for three different stress levels ( $\sigma_1 > \sigma_2 > \sigma_3$ ). The crack size-time predictions in the AL-AU range are indicated in Fig. 12. The analytical crack growth program is used to make crack size-time predictions for crack size where LEFM principles apply. Procedures and assumptions for the analytical crack growth analysis are assumed to be the same as those used for a typical damage tolerance analysis.

4. Crack size-time predictions (i.e.,  $a(t)$  versus  $t$ ) in the AL-AU range for a given stress region, depicted in Fig. 12(b), can be used to determine the SCGMC as follows: An expression for the SCGMC can be obtained from the "EIFS master curve," given in Eq. 4, by substituting  $y_{1i}(\tau) = x = \text{EIFS}$  and  $\tau = t$  to obtain Eq. 19.  $y_{1i}(\tau)$  is an EIFS in the EIFSD corresponding to a crack size  $x_1$  at time  $\tau$  in the  $i$ th stress region.

$$y_{1i}(\tau) = x_1 \exp(-Q_i \tau) \quad (19)$$





(a) Tune ("Curve Fit") the analytical crack growth Program to the EIFS master curve for the baseline case in the AL-AU range

(b) Predict  $a(t)$  vs  $t$  in AL-AU range using tuned crack growth program; obtain SCGMC for each condition using crack growth Model

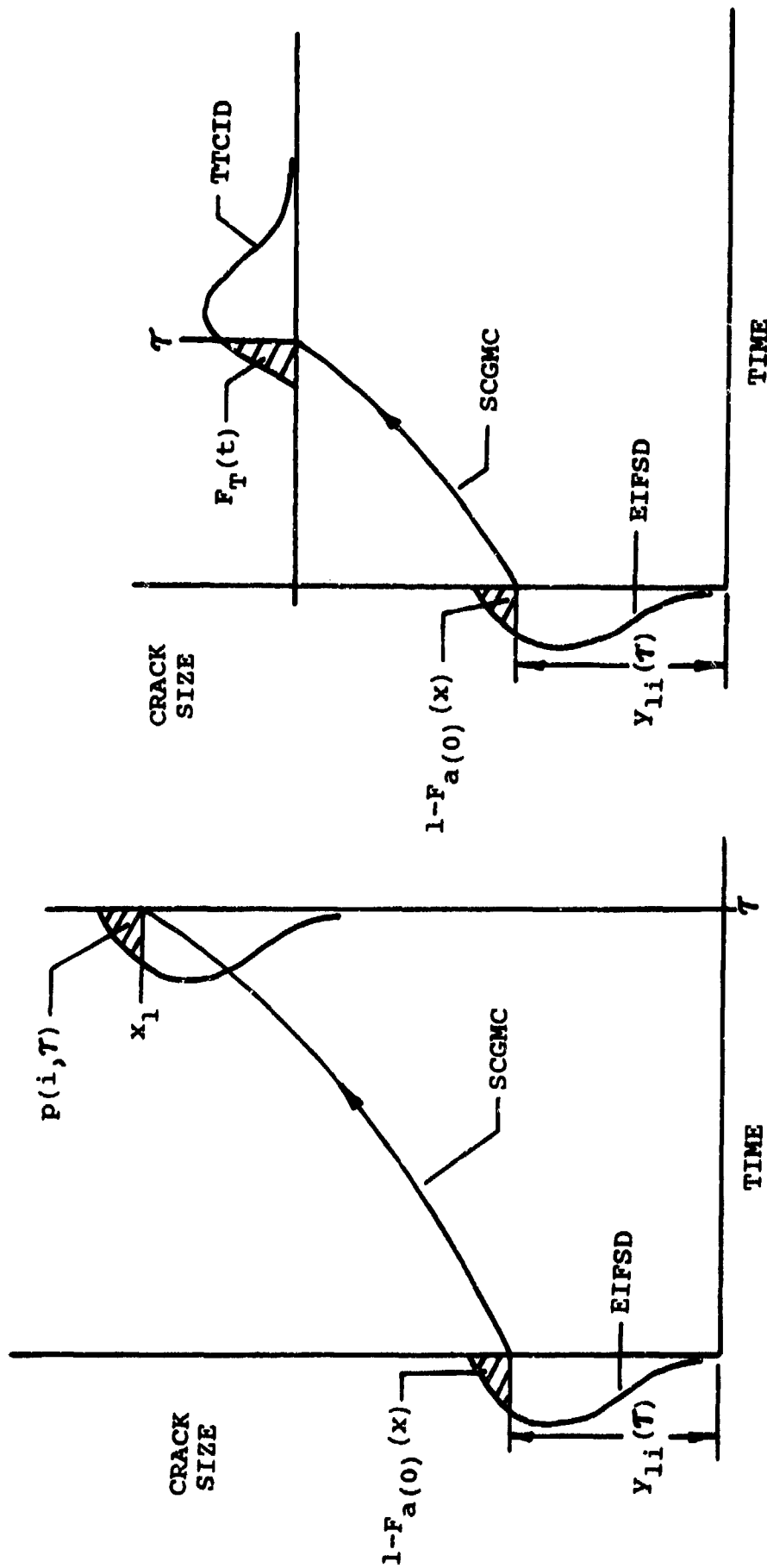
Figure 12. Concept for Determining SCGMC Using Analytical Crack Growth Program and Crack Growth Model.

In Eq. 19, the crack growth rate parameter  $Q$ , an empirical constant, can be determined using the crack size-time predictions for the AL-AU range as shown in Fig. 12(b). Methods for estimating " $Q$ " are given in Appendix C. As shown in Fig. 12(b), LEFM principles are used only for the crack size range where such principles apply.

#### 2.4.2 Probability of Crack Exceedance and Correlative Distribution of TTCI

Once the SCGMC has been determined, consistent with the basis for the EIFSD used, the EIFSD is grown forward using the applicable SCGMC (based on DCGA) to predict the probability of crack exceedance,  $p(i, \tau)$ , at a given time and the cumulative distribution of TTCI,  $F_T(t)$ , for a given reference crack size. Such predictions are conceptually described in Fig. 13. The SCGMC defines the initial flaw size,  $y_{1i}(\tau)$ , which grows to a crack size  $x_1$  at time  $\tau$  under the desired design conditions. Given  $y_{1i}(\tau)$  and the EIFSD function (e.g., Weibull compatible, lognormal compatible, two-parameter Weibull, lognormal, etc.) predictions for  $p(i, \tau)$  and  $F_T(t)$  can then be made as described in Fig. 13. The position of  $y_{1i}(\tau)$  in the EIFSD determines the resulting  $p(i, \tau)$  and/or  $F_T(t)$ .

The concept for determining  $p(i, \tau)$  and  $F_T(t)$  is very general and can be applied to different EIFSD functions and SCGMC. For the Weibull compatible EIFSD function,  $p(i, \tau)$  and  $F_T(t)$  can be computed using Eqs. 6 and 7, respectively. In Eq. 6,  $x_1$  is a variable and  $\tau$  is a fixed value for  $p(i, \tau)$  predictions. In a similar manner, in Eq. 7,  $x_1$  is a fixed value (i.e., reference crack size for TTCIs) and  $t$  is a variable for  $F_T(t)$  predictions.



(a) Probability of Crack Exceedance,  $p(i, \tau)$  (b) Cumulative Distribution of  $TTLI$ ,  $F_T(t)$

Figure 13. Description of Predictions for  $p(i, \tau)$  and  $F_T(t)$ .

## 2.5 DURABILITY ANALYSIS SOFTWARE

Computer software has been developed for an IBM PC for implementing the advanced durability analysis methodology described in this section. This includes software for: (1) storing fractographic data on floppy disks, (2) screening and plotting the fractographic data for a selected data set, (3) optimizing the EIFSD parameters for one or more fractographic data sets, (4) plotting and checking the goodness-of-fit of  $p(i, \tau)$ ,  $F_a(t)(x)$  and  $F_T(t)$  predictions for a given EIFSD. Software is also available for implementing the durability analysis extension described in Section III.

The durability analysis computer software, programmed in "GWBASIC", can be implemented on an IBM compatible PC. A "user's guide" is available for implementing the computer software in Volume V [6]. This software was used extensively in Volume II[3] to evaluate and refine the advanced durability analysis methods developed in this Volume (I).

## SECTION III

### DERIVATION OF CRACK EXCEEDANCE PROBABILITY, $p(i,t)$ , FOR DURABILITY ANALYSIS EXTENSION

The main objective of the durability analysis extension is to determine the probability of crack exceedance in the large crack size region to cover the functional impairment due to fuel leakage, ligament breakage, etc. The general procedure is to compute the distribution of crack size  $a(\tau)$  at any service time  $\tau$  from the distribution of EIFS through the transformation of the service crack growth damage accumulation equation. Hence, the main issue is to establish the accurate EIFS distribution and a service crack growth behavior. The determinations of EIFS distribution have been described in detail previously. In this section we will emphasize the accurate description of the service crack growth behavior and the crack exceedance prediction in the large crack size region.

The durability analysis methodology is simplified tremendously if the service crack growth damage accumulation can be modeled or approximated analytically. Depending on the treatment of the crack growth damage in service, the following three approaches are investigated (1) deterministic crack growth approach (DCGA), (2) stochastic crack growth approach (SCGA), and (3) two-segment deterministic-stochastic crack growth approach. Each approach will be described in the following for predicting the probability of crack exceedance in the large crack size region.

#### 3.1 SUMMARY OF EIFS DISTRIBUTIONS

Three possible distribution functions for the equivalent initial flaw size (EIFS) are: (1) Weibull compatible, (2)

lognormal and (3) two-parameter Weibull, respectively, as given in the following.

#### Weibull Compatible EIFSD Function

The Weibull compatible distribution function, proposed by Yang and Manning [13,18], is given in Eq. 20,

$$F_{a(0)}(x) = \exp\left\{-\left[\frac{\ln(x_u/x)}{\epsilon}\right]^{\alpha}\right\} \quad ; \quad 0 \leq x \leq x_u \quad (20)$$

$$= 1 \quad ; \quad x > x_u$$

where  $F_{a(0)}(x)$  = cumulative distribution of EIFS,  $x_u$  is the upper bound of the EIFS which is related to the reference crack size,  $a_0$ , and the corresponding lower bound,  $\epsilon$ , of the time-to-crack-initiation (TTCI) as given in Eq. 21.

$$x_u = a_0 e^{-Q\epsilon} \quad (21)$$

#### Lognormal EIFSD Function

$$F_{a(0)}(x) = \Phi\left(\frac{\ln x - \mu}{\sigma}\right) \quad ; \quad x \geq 0 \quad (22)$$

$$= 0 \quad ; \quad x < 0$$

In Eq. 22,  $F_{a(0)}(x)$  = cumulative distribution of EIFS,  $\Phi(\ )$  is the standardized normal distribution function, and  $\mu$  and  $\sigma$  are the mean value and standard deviation of  $\ln a(0)$ , respectively, where  $a(0)$  = EIFS.

#### Two-Parameter Weibull distribution

$$F_{a(0)}(x) = 1 - e^{-(x/\beta_0)^{\alpha_0}} \quad ; \quad x \geq 0 \quad (23)$$

$$= 0 \quad ; \quad x < 0$$

In Eq. 23,  $F_{a(0)}(x)$  = cumulative distribution of EIFS,  $\alpha_0$  and  $\beta_0$  are the shape parameter and the scale parameter of EIFS,  $a(0)$ , respectively.

### 3.2 DETERMINISTIC CRACK GROWTH APPROACH

In this approach the equivalent initial flaw sizes (EIFSs) are determined by back-extrapolating fractographic data in the crack size range of time-to-crack-initiation (TTCI), such as AL-AU, using a deterministic crack growth rate equation. Such EIFS values are referred to as "deterministic-based EIFSs." The statistical variability of the fractographic data involves the inherent statistical dispersion of the generic EIFS values as well as that of the crack growth rate. Hence, the deterministic-based EIFS values, in fact, also include the statistical variability of the crack growth rate. Consequently, when the deterministic-based EIFS distribution is grown forward under the expected service load spectra, a deterministic crack growth rate model must be used up to the crack size range of TTCI to be consistent with the fractographic test results. Hence, the deterministic crack growth approach for the durability analysis described in this section will be based on the deterministic-based EIFS distribution. The durability analysis approach is schematically shown in Fig. 14.

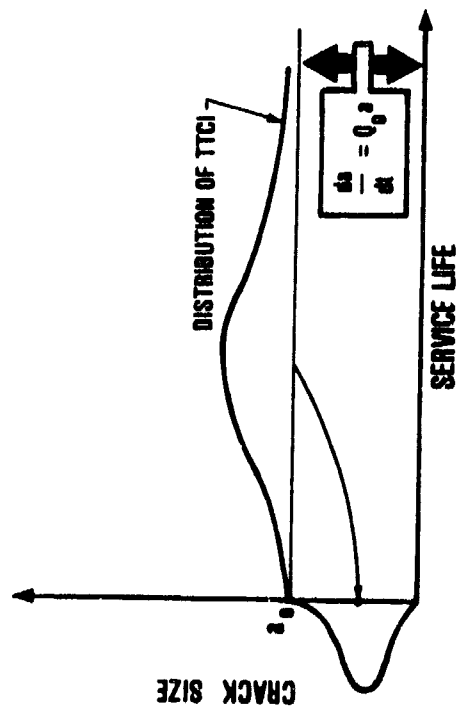
#### 3.2.1 One-Segment Deterministic Crack Growth Approach

In this approach, the entire service crack growth master curve (SCGMC) is fitted by either one of the following simple but versatile crack growth rate models recommended by Yang and Manning [13,18],

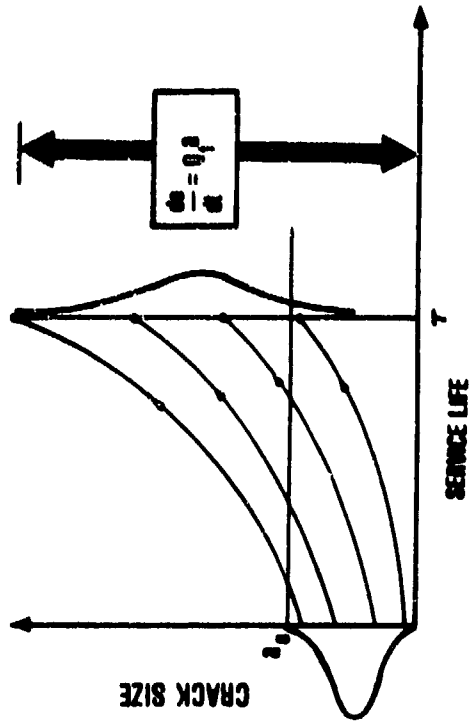
$$da/dt = Q_1(a)^{b_1} \quad (24)$$

or

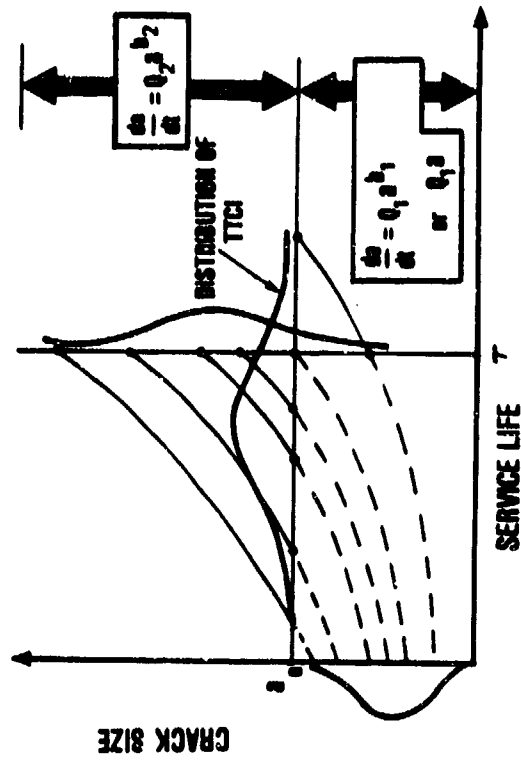
$$da/dt = Q_1 a \quad (25)$$



(a) Deterministic-Based EIFS Distribution



(b) One-Segment Deterministic Crack Growth Approach



(c) Two-Segment Deterministic Crack Growth Approach

Figure 14. Deterministic Crack Growth Approach for Durability Analysis Extension.



While Eqs. 24 and 25 appear to be simple, extensive test results [16, 58], including 3 inch wide 7475-T7351 aluminum fastener hole specimens under bomber or fighter load spectra [4], indicate that they are quite reasonable [13,18,2,19,21, 23-30,42]. Moreover, Eq. 24 has been shown to be a reasonable model for crack growth damage accumulation in critical locations, such as bolt holes, cooling air holes, etc., of gas turbine engine disks made of nickel-based superalloys[40,41].

Integrating Eq. 25 from  $t = 0$  to any service time  $t = \tau$ , one obtains

$$a(0) = a(\tau) e^{-Q_1 \tau} \quad (26)$$

The distribution function of the crack size  $a(\tau)$  can be derived from that of the EIFS,  $a(0)$ , given by Eqs. 20 - 23 through the transformation of Eq. 26 as follows:

$$\begin{aligned} F_{a(\tau)}(x) &= P[a(\tau) \leq x] = P[a(0) \leq x e^{-Q_1 \tau}] \\ &= F_{a(0)}(x e^{-Q_1 \tau}) \end{aligned} \quad (27)$$

The probability that the crack size,  $a(\tau)$ , at any service time  $\tau$  in the  $i$ th stress region will exceed any value  $x_1$ , referred to as the probability of crack exceedance, is obtained as

$$p(i, \tau) = P[a(\tau) > x_1] = 1 - F_{a(\tau)}(x_1) \quad (28)$$

Substituting Eq. 27 into Eq. 28 yields

$$p(i, \tau) = 1 - F_{a(0)}[y(x_1; \tau)] \quad (29)$$

where  $F_{a(0)}(x)$  is the distribution function of EIFS given by Eq. 20, 22 or 23 and

$$y(x_1; \tau) = x_1 e^{-Q_1 \tau} \quad (30)$$

When the more general crack growth rate equation given by Eq. 24 can be used to approximate the SCGMC with sufficient accuracy, the integration result can be expressed as follows:

$$a(\tau) = \frac{a(0)}{[1 - [a(0)]^{c_1} c_1 Q_1 \tau]^{1/c_1}} \quad (31)$$

in which

$$c_1 = b_1 - 1 \quad (32)$$

Then, the crack exceedance probability,  $p(i, \tau)$ , is given by Eq. 29, in which  $y(x_1; \tau)$  becomes

$$y(x_1; \tau) = [(x_1)^{-c_1} + c_1 Q_1 \tau]^{-1/c_1} \quad (33)$$

The one-segment deterministic crack growth approach described above is schematically shown in Fig. 14(b).

### 3.2.2 Two-Segment Deterministic Crack Growth Approach

In some cases, the service crack growth master curve (SCGMC) may not be fitted by a single equation, e.g., Eq. 24 or 25, with sufficient accuracy. Then, the SCGMC may be separated into two regions; one with the crack size smaller than the reference crack size,  $a_0$ , at crack initiation, and the other with the crack size larger than  $a_0$ .

In the crack size region smaller than  $a_0$ , the following two equations, identical to Eqs. 24 and 25, can be used

$$da/dt = Q_1(a)^{b_1} \quad \text{for } a \leq a_0 \quad (34)$$

or

$$da/dt = Q_1 a \quad \text{for } a \leq a_0 \quad (35)$$

In the second region where the crack size is larger than  $a_0$ , Eq. 36 can be used for approximation

$$da/dt = Q_2(a)^{b_2} \quad \text{for } a > a_0 \quad (36)$$

in which  $b_2$  may be equal to unity. Such an approach is schematically shown in Fig. 14(c). The crack growth rate parameters  $(Q_1, b_1)$  in the small crack size region are, in general, different from  $(Q_2, b_2)$  in Eq. 36.

Let the time-to-crack-initiation for the reference crack size  $a_0$  be denoted by  $T$ , i.e.,  $a(T) = a_0$ . When the crack size of interest,  $x_1$ , is smaller than  $a_0$ , the crack exceedance probability,  $p(i, \tau)$ , is identical to that given by Eqs. 29 and 30. It is mentioned that the service crack growth master curve (SCGMC) in this region should be "tuned" or "curve fitted" to be consistent with the EIFS master curve and then best-fitted by Eq. 34 or 35.

When the crack size of interest  $x_1$  is greater than  $a_0$ , the crack size  $a(\tau)$  at any service time  $\tau > T$  can be obtained as follows. Integrating Eq. 35 from  $t = 0$  to  $t = T$  (at which  $a(T) = a_0$ ) leads to the following expression

$$T = Q_1^{-1} \ln[a_0/a(0)] \quad (37)$$

In the region where  $a(\tau) > a_0$  (or  $\tau > T$ ), Eq. 36 is integrated with  $b_2 = 1$  from  $t = T$  to  $t = \tau$ ; with the result

$$\tau = T + Q_2^{-1} \ln[a(\tau)/a_0] \quad ; \quad \text{for } a(\tau) > a_0 \quad (38)$$

Equating Eqs. 37 and 38, one obtains the relationship between the crack sizes  $a(\tau)$  and  $a(0)$  as follows

$$a(0) = [a(\tau)]^{Q_1/Q_2} \exp(\Lambda - Q_1 \tau) \quad (39)$$

in which

$$\Lambda = [1 - (Q_1/Q_2)] \ln a_0 \quad (40)$$

Thus the distribution function,  $F_a(\tau)(x)$ , of the crack size  $a(\tau)$  can be obtained from that of  $a(0)$  through the transformation of Eq. 39, and the probability of crack exceedance,  $p(i, \tau)$ , can be obtained in a similar manner as Eq. 29, i.e.,

$$p(i, \tau) = 1 - F_{a(0)}[y(x_i; \tau)] \quad (41)$$

in which

$$y(x_i; \tau) = (x_i)^{Q_1/Q_2} \exp(\Lambda - Q_1 \tau) \quad (42)$$

where  $\Lambda$  is given by Eq. 40.

The solution obtained above holds for  $b_1 = b_2 = 1$ . For the case where  $b_1 = 1$  and  $b_2 \neq 1$ , the solution is given by Eq. 41 except that

$$\left. \begin{aligned} y(x_i; \tau) &= \exp\{\Lambda - Q_1 \tau - Q_1 (c_2 Q_2)^{-1} (x_i)^{-c_2}\} \\ \Lambda &= \ln a_0 + Q_1 (c_2 Q_2)^{-1} (a_0)^{-c_2} \\ c_2 &= b_2 - 1 \end{aligned} \right\} \quad (43)$$

Furthermore, for the case where  $b_1 \neq 1$  and  $b_2 \neq 1$ , the crack exceedance probability is also given by Eq. 41 in which

$$\left. \begin{aligned} y(x_i; \tau) &= (x_i)^{c_2/c_1} / \left[ \frac{c_1 Q_1}{c_2 Q_2} + (x_i)^{c_2} c_1 Q_1 (\Lambda + \tau) \right]^{1/c_1} \\ \Lambda &= (c_1 Q_1)^{-1} (a_0)^{-c_1} - (c_2 Q_2)^{-1} (a_0)^{-c_2} \\ c_1 &= b_1 - 1, \quad c_2 = b_2 - 1 \end{aligned} \right\} \quad (44)$$

### 3.2.3 General Deterministic Crack Growth Approach

The service crack growth master curve (SCGMC) can be obtained numerically using an analytical crack growth computer program, [e.g., 38,39]. Note that the SCGMC varies from one stress region to another. When the SCGMC cannot be approximated by two or more segments of analytical models (equations) with sufficient accuracy, the numerically defined SCGMC can be used directly for the durability analysis. The solution for the crack exceedance probability,  $p(i,T)$ , can be obtained numerically in the following manner [13,19].

The SCGMC can be represented symbolically by the following form

$$a(t_1) = y[a(t_2); t_2 - t_1] \quad (45)$$

in which the crack size,  $a(t_1)$ , at  $t_1$  is expressed as a function of the crack size,  $a(t_2)$ , at  $t_2$  with  $t_2 > t_1$  and the time difference  $t_2 - t_1$ . The function  $y$  is a monotonically increasing function of  $a(t_2)$  and  $t_2 - t_1$ .

Strictly speaking, this relation may not be valid for the general case within one flight of a particular mission because of the load sequence effect and other contributors. Thus, within one flight of a particular mission, the relationship between  $a(t_1)$  and  $a(t_2)$  depends on both  $T_1$  and  $t_2$ . Since, however, the design loading spectra in one lifetime consists of many repeated missions and flights, it is reasonable to assume that the relationship between  $a(t_1)$  and  $a(t_2)$  depends only on the time difference  $t_2 - t_1$  of the service time. Such an expedient approximation appears to be acceptable.

For the special case in which  $t_1 = 0$  and  $t_2 = t$ , Eq. 45 becomes

$$a(0) = y[a(\tau); \tau] \quad (46)$$

The symbolic equation, such as Eq. 45 or 46, will serve, mathematically, as a transfer function for transforming the statistical distribution of the crack population from one service time (such as  $t = 0$ ) to another.

The distribution function of the crack size  $a(\tau)$  at any service time  $\tau$  can be derived from that of  $a(0)$  through the transformation of Eq. 46 as follows

$$F_{a(\tau)}(x) = P[a(\tau) \leq x] = P[a(0) \leq y(x; \tau)] = F_{a(0)}[y(x; \tau)]$$

or

(47)

$$F_{a(\tau)}(x) = F_{a(0)}[y(x; \tau)]$$

in which the property that  $y(x; \tau)$  is a monotonically increasing function has been used. The probability of crack exceedance,  $p(i, \tau)$ , is obtained as

$$p(i, \tau) = P[a(\tau) \geq x_i] = 1 - F_{a(\tau)}(x_i)$$

(48)

$$= 1 - F_{a(0)}[y(x_i; \tau)]$$

where the distribution of EIFS,  $F_{a(0)}(x)$ , is given by Eq. 20, 22, or 23. The argument  $y(x_i; \tau)$  can be determined graphically (or numerically with interpolation procedures) from the SCGMC as shown schematically in Fig. 15. The numerical procedures for the determination of  $y(x_i; \tau)$  are described as follows; (1) Given the crack size of interest,  $x_i$ , determine the corresponding time  $t_i$  from the SCGMC by interpolation, and (2) Determine the crack size  $y(x_i; \tau)$  corresponding to the time  $t_i - \tau$  from the SCGMC again using the interpolation procedures (Fig. 15).

The computation of the crack exceedance probability,  $p(i, \tau)$ , described above is straight-forward and simple.

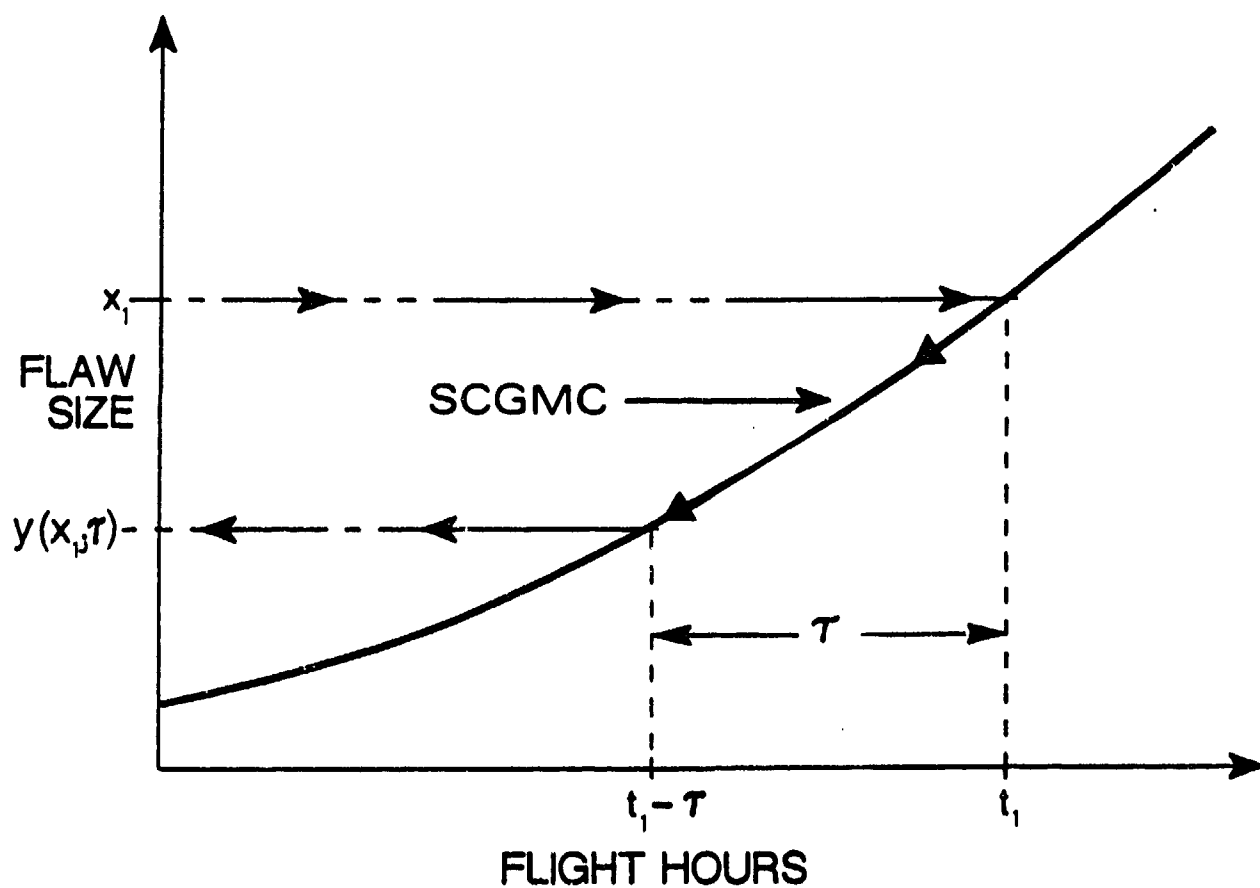


Figure 15. General Deterministic Crack Growth Approach for Service Crack Growth Master Curve.

### 3.3 STOCHASTIC CRACK GROWTH APPROACH

Analysis results of fractographic data indicate that the crack growth damage accumulation in the small crack size region involves considerable statistical variability [e.g., 27 42-44]. Theoretically, the statistical variability of the fractographic data in the crack size region close to TTCI or the TTCI fractographic data is contributed by the statistical dispersions of both the equivalent initial flaw size and the crack growth damage accumulation up to  $a_0$ . In the deterministic crack growth approach, the distribution of the EIFS is determined by back-extrapolating the fractographic data using a deterministic crack growth model. The statistical dispersion of the EIFS obtained also includes the statistical dispersion of the crack growth rate in the small crack size region. Such an approach is quite reasonable as long as the deterministic crack growth rate model is used to grow the EIFS values up to the reference crack size  $a_0$ . Such an expedient approach simplifies the durability analysis methodology tremendously, and the simplification makes the proposed durability analysis very attractive.

Another approach [28,29] is to treat the crack growth rate as a stochastic process, and the EIFS values are obtained by back-extrapolating the fractographic data stochastically as shown in Fig. 16(a). This implies that the statistical variability of the crack growth rate is filtered out in the back-extrapolation process. The EIFS values thus obtained are referred to as "stochastic-based EIFSs." Then, under the expected service load spectra, the forward crack propagation from EIFS must be stochastic. Such an approach will be addressed in this section and the stochastic-based EIFS will be used for the computation of the crack exceedance probability.



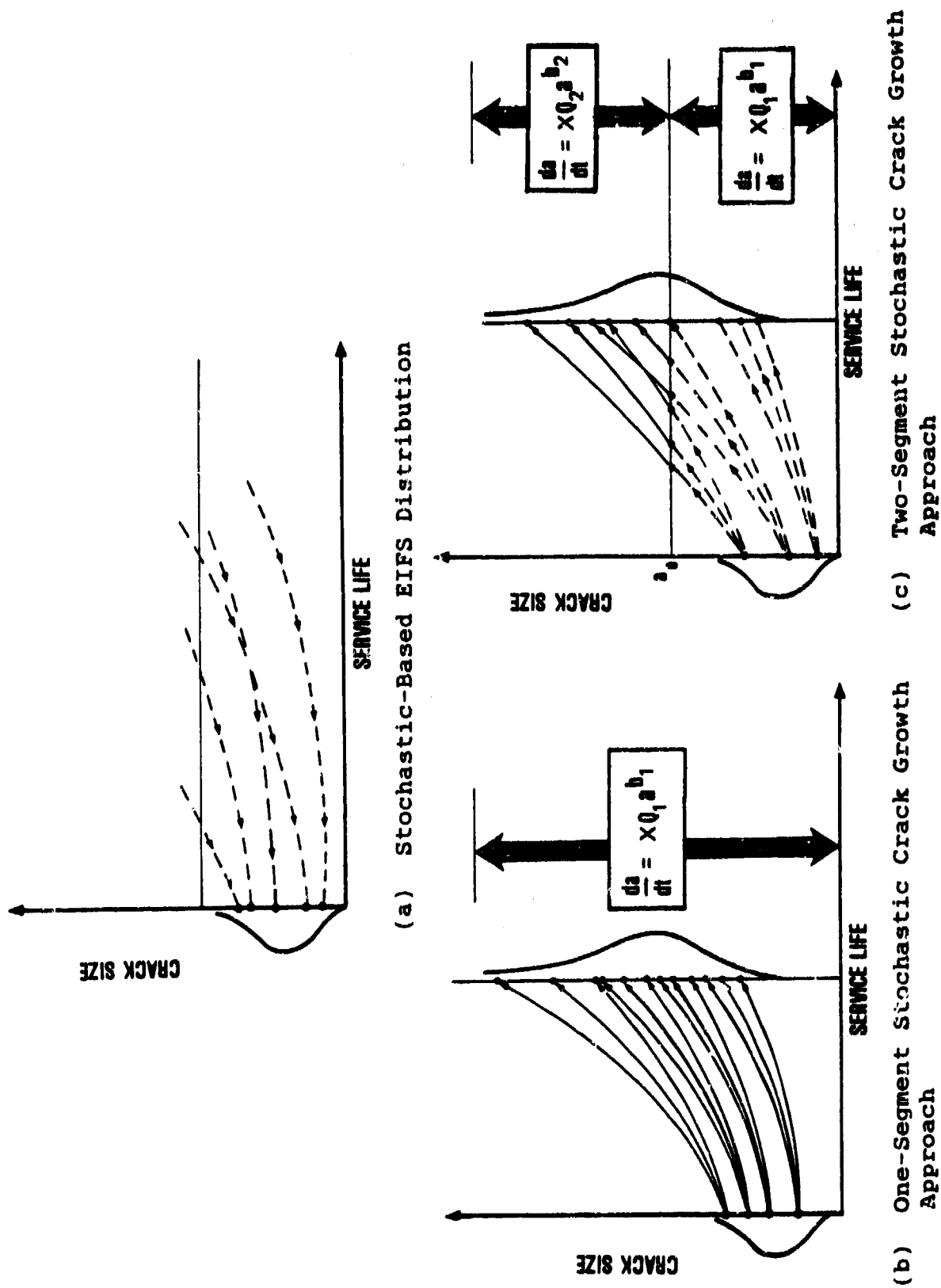


Figure 16. Stochastic Crack Growth Approach for Durability Analysis Extension.

The stochastic crack growth approach depends on a simple, yet reasonable, stochastic crack growth rate model. Recently, a simple and plausible stochastic crack growth model proposed by Yang, et al [27,40-42,45-51] can be used conveniently for durability analysis as follows

$$da/dt = XQ_1(a)^{b_1} \quad (49)$$

in which  $X$  is a lognormal random variable with a median value equal to unity, i.e.,  $X$  takes values around unity. Thus, the deterministic crack growth rate model given by Eq. 24 represents the median crack growth behavior, whereas the statistical variability of the crack growth rate is taken care of by the lognormal random variable  $X$ . Such a stochastic crack growth model is referred to as the lognormal random variable model [27,50,51]. The stochastic crack growth approach for the durability analysis is schematically shown in Fig. 16.

While many different stochastic crack growth models have been suggested in the literature [e.g., 43,44,52-56], the lognormal random variable model, Eq. 49, is the simplest stochastic model most suitable for practical applications, such as the durability analysis. It has been demonstrated [e.g., 27,50,51] that Eq. 49 can be used satisfactorily for the fatigue crack propagation in 7475-T7351 fastener holes under bomber and fighter loading spectra in both the small crack size and large crack size regions.

Since  $X$  is a lognormal random variable with a median value equal to unity, the random variable

$$Z = \log X \quad (50)$$

is normal with zero mean and standard deviation  $\sigma_z$ . The probability density function of  $X$  is given by

$$f_X(u) = \frac{\log e}{\sqrt{2\pi} \times \sigma_2} \exp \left\{ -\frac{1}{2} \left( \frac{\log u}{\sigma_2} \right)^2 \right\} \quad ; \quad u \geq 0$$

$$= 0 \quad ; \quad u < 0 \quad (51)$$

### 3.3.1 One-Segment Stochastic Crack Growth Approach

Suppose the stochastic service crack growth master curve can be represented by Eq. 49. Then, integrating Eq. 49 from  $t = 0$  to  $t = \tau$ , for  $b_1 = 1$ , one obtains

$$a(\tau) = a(0)e^{XQ_1\tau} \quad (52)$$

or

$$a(0) = a(\tau)e^{-XQ_1\tau} \quad (53)$$

The crack size,  $a(\tau)$ , at any service time  $\tau$  is a function of two random variables,  $a(0)$  and  $X$ . To derive the distribution function of  $a(\tau)$ , a particular value  $u$  is assigned to  $X$ , i.e.,  $X = u$ , and then the theorem of total probability is used. The conditional distribution function of  $a(\tau)$  given  $X = u$  is derived as follows

$$F_{a(\tau)}(x_1 | X=u) = P[a(\tau) \leq x_1 | X=u] \quad (54)$$

$$= P[a(0) \leq y(x_1; \tau | X=u)] = F_{a(0)}[y(x_1; \tau | X=u)]$$

in which it follows from Eq. 53 that

$$y(x_1; \tau | X=u) = x_1 e^{-uQ_1\tau} \quad (55)$$

The conditional crack exceedance probability, denoted by  $p(i, \tau | X=u)$ , given  $X = u$ , is obtained as

$$p(i, \tau | X=u) = 1 - F_{a(\tau)}(x_1 | X=u)$$

$$= 1 - F_{a(0)}[y(x_1 | X=u)] \quad (56)$$

in which Eq. 54 has been used.

The unconditional crack exceedance probability is obtained from the conditional one using the theorem of total probability as

$$\begin{aligned} p(l, \tau) &= \int_0^{\infty} p(l, \tau | X=u) f_X(u) du \\ &= 1 - \int_0^{\infty} F_a(0)[y(x_1; \tau | X=u)] f_X(u) du \end{aligned} \quad (57)$$

in which  $y(x_1; \tau | X=u)$  and  $f_X(u)$  are given by Eqs. 55 and 51, respectively.

When  $b_1 \neq 1$ , Eq. 49 can be integrated analytically and the crack exceedance probability,  $p(l, \tau)$ , is identical to that presented in Eq. 57, except that  $y(x_1; \tau | X=u)$  is given by

$$y(x_1; \tau | X=u) = \frac{x_1}{[1 + (x_1)^{c_1} c_1 Q_1 \tau]^{1/c_1}} \quad (58)$$

in which  $C_1 = b_1 - 1$ .

The solution derived above, Eq. 57, is expressed in terms of the distribution function of EIFS,  $F_a(0)(x)$ , that is given by Eq. 20 in the case of the Weibull compatible distribution, by Eq. 22 in the case of lognormal distribution, and by Eq. 23 in the case of the two-parameter Weibull distribution. For instance, when the Weibull compatible distribution is used, the crack exceedance probability given by Eq. 57 becomes

$$p(l, \tau) = 1 - \int_0^{\infty} \exp \left\{ - \left[ \frac{\ln \left[ \frac{x_u}{y(x_1; \tau | X=u)} \right]}{\phi} \right]^a \right\} f_X(u) du \quad (59)$$

In the numerical computation using Eq. 59, it should be noted that the term in the integrand  $\exp(-[\ln(x_u/y(x_1; \tau | X=u))/\phi]^a) = 1$  if  $y(x_1; \tau | X=u) > x_u$ .

The crack exceedance probability presented in Eq. 57 should be evaluated numerically, since the analytical integration, in general, is not feasible. However, the numerical integration is simple and straight-forward. The one-segment stochastic crack growth approach is schematically shown in Fig. 16(b).

### 3.3.2 Two-Segment Stochastic Crack Growth Approach

When the service crack growth master curve cannot be approximated by a single crack growth rate equation, Eq. 49, with sufficient accuracy, a two-segment crack growth approximation analogy to that described in Section 3.2.2 should be used.

In the crack size region smaller than the reference crack size  $a_0$ , the crack growth rate equation is approximated by

$$da/dt = XQ_1(a)^{b_1} \quad ; \quad \text{for } a < a_0 \text{ or } t < T \quad (60)$$

In the crack size region larger than  $a_0$ , the following approximation is used

$$da/dt = XQ_2(a)^{b_2} \quad ; \quad \text{for } a > a_0 \text{ or } t > T \quad (61)$$

Integrating Eq. 60 from  $t = 0$  to  $t = T$  (at which  $a(T) = a_0$ ), with  $b_1 = 1$ , leads to the following expression

$$a(T) = a(0)e^{XQ_1 T} = a_0 \quad (62)$$

from which

$$T = \frac{1}{XQ_1} \ln \left[ \frac{a_0}{a(0)} \right] \quad (63)$$

Equation 61 can be integrated from  $T$  (i.e.,  $a_0$ ) to any service time  $T > T$ ,

$$a(\tau) = a_0 e^{XQ_2(\tau-T)} \quad (64)$$

Substituting Eq. 63 into Eq. 64 leads to the relation between  $a(\tau)$  and  $a(0)$  as follows

$$a(0) = [a(\tau)]^\gamma (a_0)^{1-\gamma} e^{-XQ_2\tau\gamma} \quad (65)$$

in which

$$\gamma = Q_1/Q_2 \quad (66)$$

When the crack size of interest,  $x_1$ , is smaller than the reference crack size  $a_0$ , i.e.,  $x_1 < a_0$ , the probability of crack exceedance is identical to Eqs. 57 and 55. When the crack size of interest,  $x_1$ , is larger than  $a_0$ , however, Eq. 65 should be used to derive the crack exceedance probability; with the result

$$p(1, \tau) = 1 - \int_0^\infty F_{a(0)}[y(x_1; \tau) | X=u] f_X(u) du ; \text{ for } x_1 > a_0 \quad (67)$$

in which

$$y(x_1; \tau | X=u) = x_1^\gamma (a_0)^{1-\gamma} e^{-uQ_2\tau\gamma} \quad (68)$$

Again, for the Weibull compatible distribution given by Eq. 20, the term  $F_{a(0)}[y(x_1; \tau | X=u)]$  in the integrand of Eq. 67 is equal to unity if  $y(x_1; \tau | X=u)$  is greater than  $x_u$ , i.e.,  $F_{a(0)}[y(x_1; \tau | X=u)] = 1$  for  $y(x_1; \tau | X=u) > x_u$ .

When  $b_1$  and  $b_2$  in Eqs. 60 and 61 are not equal to 1.0, i.e.,  $b_1 \neq 1$  and  $b_2 \neq 1$ , the crack exceedance probability derived in Eq. 67 is still valid except that  $y(x_1; \tau | X=u)$  is different; however, it can be derived easily. The two-segment stochastic crack growth approach is schematically shown in Fig. 16(c).

### 3.3.3 General Stochastic Crack Growth Approach

When the service crack growth master curve cannot be approximated by two or more segments of stochastic crack growth rate equations with sufficient accuracy, numerically defined general stochastic service crack growth master curves should be used. The derivations and numerical procedures for computing the crack exceedance probability are similar to those described in Section 3.2.3. The only difference is that the crack exceedance probability obtained in Section 3.2.3 corresponds to a particular percentile with certain probability, and a sufficient number of repetitive computations should be made to cover a wide range of percentiles.

### 3.4 TWO-SEGMENT DETERMINISTIC-STOCHASTIC CRACK GROWTH APPROACH

Two approaches for establishing the equivalent initial flaw size have been considered; namely, the deterministic-based EIFS and the stochastic-based EIFS. For the deterministic-based EIFS, a deterministic crack growth rate model should be applied to grow the entire EIFS population up to the reference crack size  $a_0$ , whereas a stochastic crack growth rate model should be used to grow the stochastic-based EIFS population. It has been shown [28,29] that both approaches are reasonable for durability analysis. However, the deterministic-based EIFS concept appears to be more attractive for two reasons: (1) The mathematical expressions and computations are much simpler, and (2) within the short crack size range, e.g., crack size smaller than 0.005", the crack growth behavior of short cracks is not well understood and the applicability of linear elastic fracture mechanics is not certain. Thus, the EIFS is simply a hypothetical concept of convenience for conducting various types of analyses, such

as the durability analysis. Hence, it is not absolutely necessary to use the stochastic-based EIFS concept for the purpose of practical applications.

Because of simplicity in the durability analysis procedures, the deterministic-based EIFS concept will be considered in this section, and a deterministic crack growth rate model

$$da/dt = Q_1(a)^{b_1} \quad ; \quad \text{for } a < a_0 \text{ or } t < T \quad (69)$$

will be used to grow the deterministic-based EIFS population up to the reference crack size  $a_0$  to be consistent with the fractographic results.

When the crack size of interest is extended beyond the reference crack size  $a_0$ , such as that associated with the functional impairment, the effect of the statistical variability of the crack growth rate influences the durability analysis results. In particular, such an effect on the prediction of the crack exceedance probability must be accounted for. As a result, the stochastic crack growth rate model described previously will be employed to grow the cracks in the crack size region larger than the reference crack size  $a_0$ , i.e.,

$$da/dt = XQ_2(a)^{b_2} \quad ; \quad \text{for } a > a_0 \quad (70)$$

in which  $X$  is a lognormal random variable with a median value equal to unity. The approach described above is referred to as the two-segment deterministic-stochastic approach [30].

Integrating Eq. 69 from  $t = 0$  to  $t = T$  for  $b_1 = 1$ , one obtains

$$T = Q_1^{-1} \ln[a_0/a(0)] \quad (71)$$



in which it is understood that  $a(T) = a_0$ .

In the region where  $a(\tau) > a_0$  (or  $\tau > T$ ), Eq. 70 is integrated with  $b_2 = 1$  from  $t = T$  to  $t = \tau$  (or from  $a(t) = a_0$  to  $a(t) = a(\tau)$ ); with the result

$$\tau = T - (XQ_2)^{-1} \ln[a(\tau)/a_0] \quad ; \quad a(\tau) > a_0 \quad (72)$$

Equating Eqs. 71 and 72 leads to the following relation between  $a(\tau)$  and  $a(0)$

$$a(0) = a_0 e^{-Q_1 \tau} [a(\tau)/a_0]^{\gamma/X} \quad \text{for } a(\tau) > a_0 \quad (73)$$

in which

$$\gamma = Q_1/Q_2 \quad (74)$$

When the crack size,  $a(\tau)$ , at any service time is smaller than  $a_0$ , the relation between  $a(\tau)$  and  $a(0)$  is obtained by integrating Eq. 69 from  $t = 0$  to  $t = \tau$ ,

$$a(0) = a(\tau) e^{-Q_1 \tau} \quad ; \quad a(\tau) < a_0 \quad (75)$$

The crack exceedance probability,  $p(i, \tau)$ , can be computed depending on the crack size of interest  $x_1$  in the following manner.

(1) When the crack size of interest  $x_1$  is smaller than the reference crack size  $a_0$ , the distribution function  $F_{a(\tau)}(x_1)$  of the crack size  $a(\tau)$  for  $x_1 < a_0$  is derived from the distribution function of  $a(0)$  through the transformation of Eq. 75,

$$F_{a(\tau)}(x_1) = F_{a(0)}[y(x_1; \tau)] \quad (76)$$

in which

$$y(x_1; \tau) = x_1 e^{-Q_1 \tau} \quad (77)$$

and the crack exceedance probability,  $p(i, \tau)$ , is given by

$$p(i, \tau) = 1 - F_{a(0)}[y(x_1; \tau)] \quad \text{for } x_1 \leq a_0 \quad (78)$$

(2) When the crack size of interest  $x_1$  is larger than  $a_0$ , the conditional distribution function of  $a(\tau)$  at any service time  $\tau$  given  $X=u$  can be derived from that of  $a(0)$  through the transformation of Eq. 73. Then, the unconditional distribution function,  $F_{a(\tau)}(x_1)$ , of  $a(\tau)$  can be obtained by using the theorem of total probability; with the result

$$F_{a(\tau)}(x_1) = \int_0^\infty F_{a(0)}[G(x_1; \tau | X=u)] f_X(u) du \quad (79)$$

in which

$$G(x_1; \tau | X=u) = a_0 e^{-Q_1 \tau} (x_1/a_0)^{\gamma/u} \quad (80)$$

The crack exceedance probability for  $x_1 > a_0$  is given by

$$p(i, \tau) = 1 - \int_0^\infty F_{a(0)}[G(x_1; \tau | X=u)] f_X(u) du \quad (81)$$

in which  $G(x_1; \tau | X=u)$  is given by Eq. 80 and  $f_X(u)$  is the probability density function of the lognormal random variable  $X$  given by Eq. 51.

When the Weibull compatible distribution, Eq. 20, is used for the EIFS, the condition that  $F_{a(0)}[G(x_1; \tau | X=u)] = 1$  for  $G(x_1; \tau | X=u) > x_u$  should be accounted for in the computer program for computing the crack exceedance probability  $p(i, \tau)$ , Eq. 81. The two-segment deterministic-stochastic crack growth approach described above is schematically shown in Fig. 17.

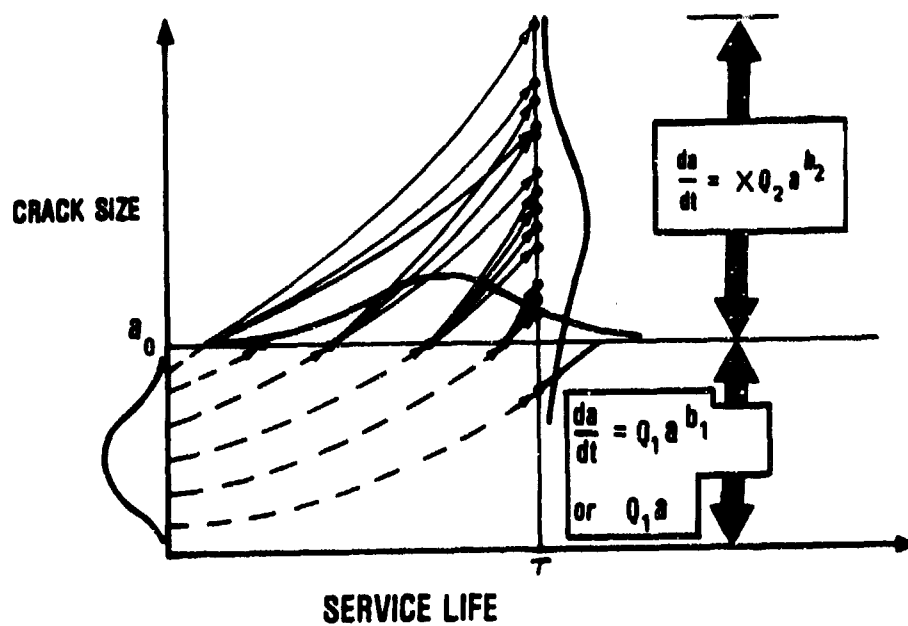


Figure 17. Two-Segment Deterministic-Stochastic Approach.

### 3.5 DISTRIBUTION OF TIME TO REACH ANY CRACK SIZE $x_1$

The determination of the crack exceedance probability,  $p(i, \tau)$ , in both the large and small crack size regions has been presented. Another quantity of interest in the durability analysis is the distribution of service time to reach any specific crack size  $x_1$ . Let  $T(x_1)$  be the time for a crack to reach any size  $x_1$  and  $F_{T(x_1)}(\tau)$  be the corresponding cumulative distribution function, i.e.,  $F_{T(x_1)}(\tau) = P[T(x_1) \leq \tau]$ . The distribution function of  $T(x_1)$  is the probability that the crack will reach a size  $x_1$  before the service time  $\tau$ . Such a probability is equal to the probability that the crack size  $a(\tau)$  at service time  $\tau$  will exceed  $x_1$ , which is simply the probability of crack exceedance. Hence,

$$F_{T(x_1)}(\tau) = P[T(x_1) \leq \tau] = P[a(\tau) \leq x_1] = p(i, \tau) \quad (82)$$

Consequently, the distribution function of the service time to reach any crack size  $x_1$  is obtained by computing the crack exceedance probability,  $p(i, \tau)$ , at different values of  $\tau$ .

### 3.6 SERVICE CRACK GROWTH MASTER CURVE FOR DURABILITY ANALYSIS EXTENSION

A general procedure for determining a suitable service crack growth master curve (SCGMC) for growing the EIFSD forward is described and discussed in this section. The SCGMC is determined using a general crack growth computer program [e.g., 38,39] and a suitable crack growth model (e.g., Eq. 35). It is not essential that a single SCGMC, composed of two segments, actually be generated to implement the durability analysis extension. The SCGMC can be represented by crack growth rate models (e.g., Eq. 34 and 35), and the expressions for  $p(i, \tau)$  and/or  $F_T(t)$  contain the applicable model parameters. However, it is important to understand how the SCGMC is developed and how it works so that the needed crack growth model parameters can be determined.

In the following the two-segment DCGA-DCGA will be emphasized and applications for the DCGA-SCGA will be discussed. The first segment covers the small crack size range (i.e.,  $0 \leq a(t) \leq AU$ ) and the second segment covers the large crack size range,  $AU - AU'$  as shown in Fig. 18.

1. A suitable analytical crack growth program [e.g., 38,39] is "tuned" or "curve fitted" to the basis for the EIFSD. This involves curve fitting the analytical crack growth program to the EIFS master curves reflected in the EIFSD, in the AL-AU range. Refer to Step 2 in Section 2.4.1.

2. Use the tuned analytical crack growth program to predict the crack growth (i.e.,  $a(t)$  versus  $t$ ) from AL to  $AU'$  for the desired analysis conditions (i.e., stress level, load spectrum, % bolt load transfer, etc.). For example, such crack growth predictions are shown in Fig. 18 for three different stress levels, in which  $\sigma_1 > \sigma_2 > \sigma_3$ . In Fig. 18 the analytical crack growth predictions cover a crack size range where LEFM principles apply (i.e.,  $a(t) \geq AL$ ).

3. The SCGMC for segment 1 (i.e.,  $0 \leq a(t) \leq AU$ ) is determined using a suitable crack growth model (e.g., Eq. 35) to fit the analytical crack growth predictions in the AL-AU range.  $Q_1$  in Eq. 35 can be determined using the procedure described in Appendix C. Similar methods for determining  $Q_1$  and  $b_1$  in Eq. 34 are available [15].

4. In a similar manner, the SCGMC for segment 2 (i.e.,  $AU \leq a(t) \leq AU'$ ) can be determined using a suitable crack growth model as depicted in Fig. 18 to fit the analytical crack growth predictions in the AL-AU range. Use the same methods described previously to determine the model parameter(s).

5. The two crack growth master curve segments for a given stress level can be physically combined into a single SCGMC as illustrated in Fig. 19. At point ① segments 1 and

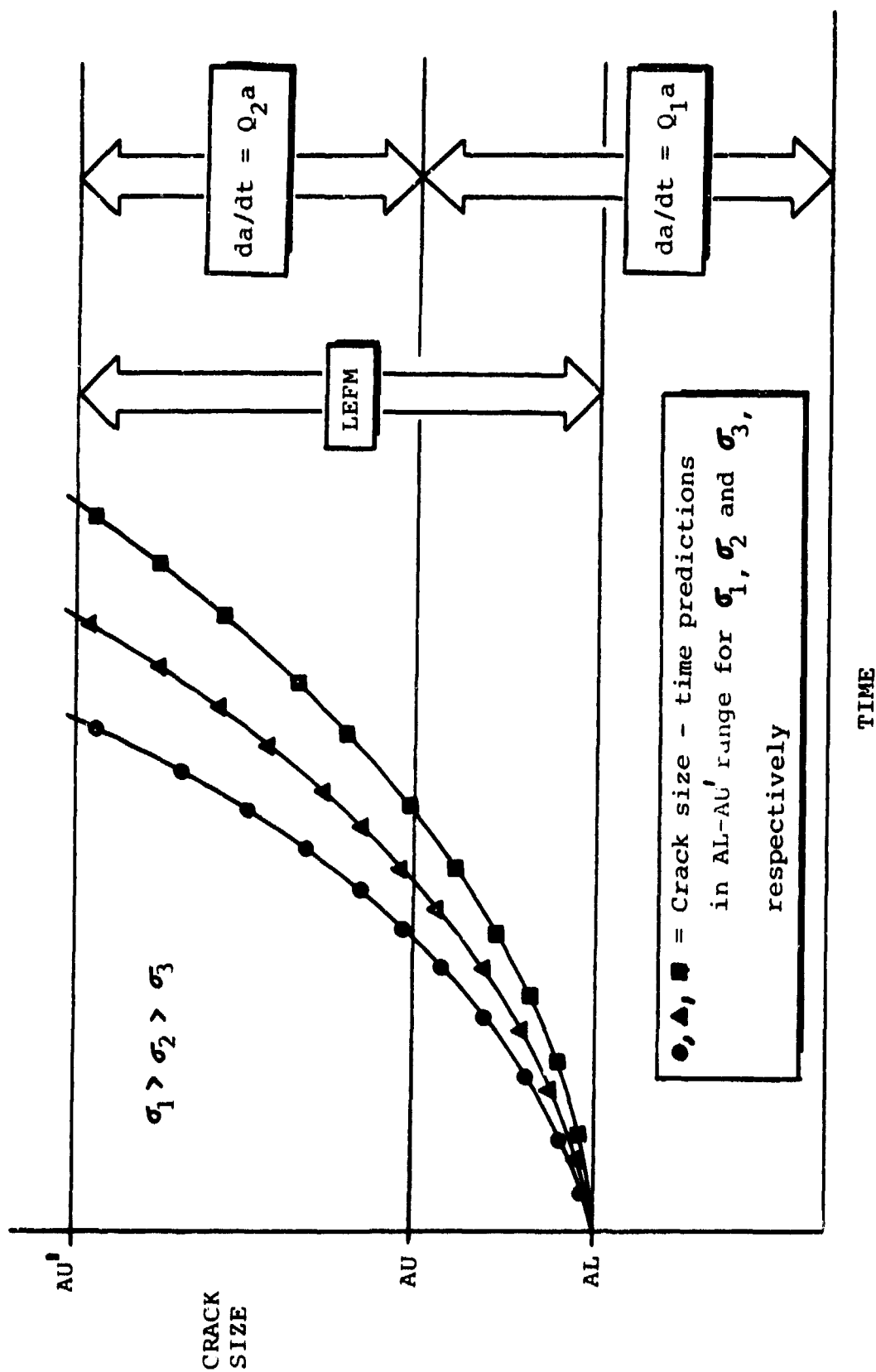


Figure 18. Use of Analytical Crack Growth Predictions and Crack Growth Model to Establish SCGMs for Durability Analysis Extension.

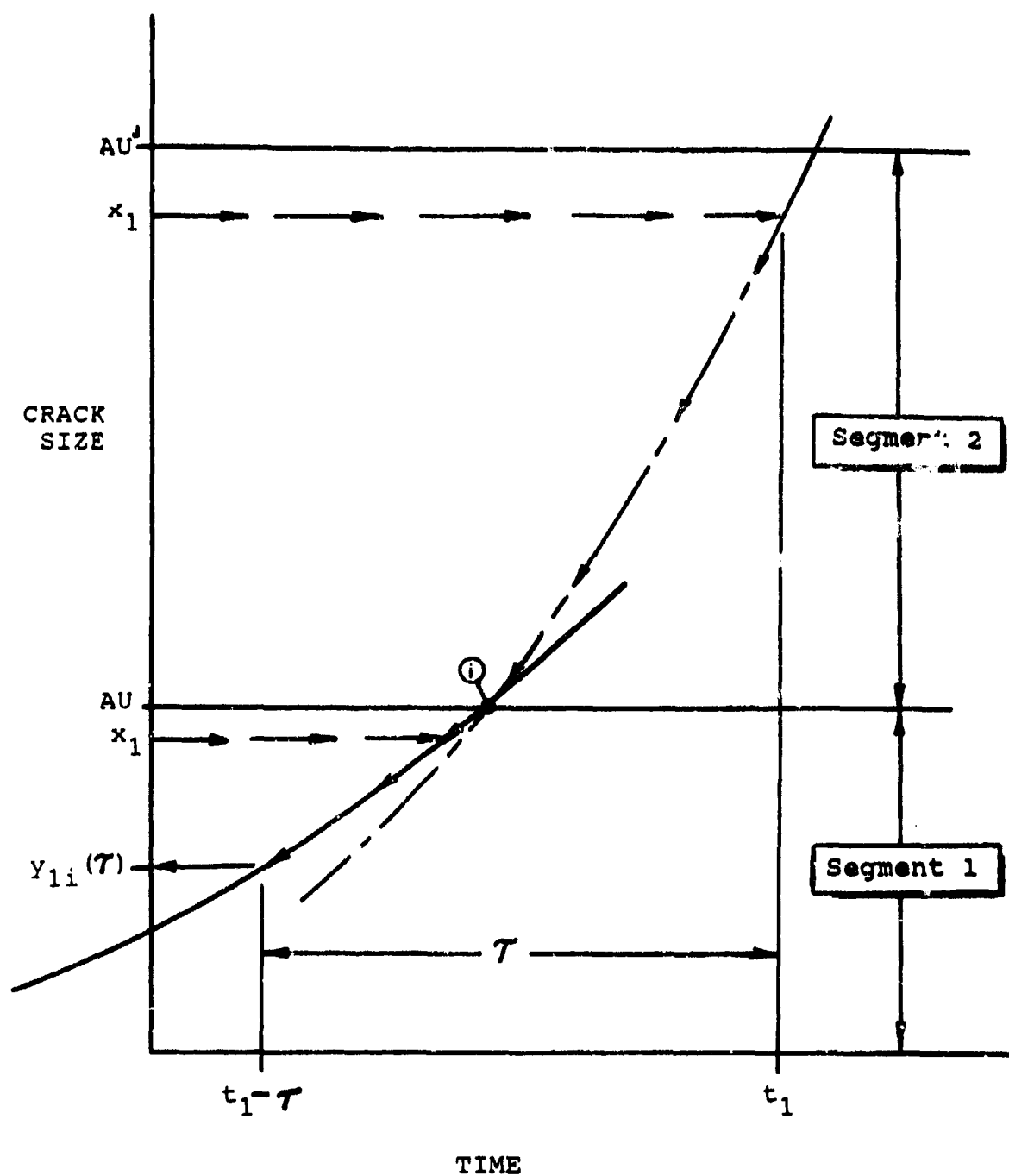


Figure 19. Mechanics of Two-Segment SCGMC for Durability Analysis Extension.

2 have the same  $a(t), t$  values but not necessarily the same slopes. The two-segment SCGMC is functionally the same as that described in Fig. 15. For example, the EIFS,  $Y_{11}(\tau)$ , corresponding to crack size  $x_1$  at any service time  $\tau$  is shown in Fig. 19.

Actually, the two-segment SCGMC for the DCGA-DCGA can also be used for the DCGA-SCGA for the same crack growth analysis conditions (e.g., stress level, load spectra, etc.). The only difference is that the parameter  $\sigma_z$  is required to implement the DCGA-SCGA.  $\sigma_z$  is the standard deviation of  $da/dt$  with respect to the plot of  $\ln da/dt = \ln a(t) + Qt$ .  $\sigma_z$  can be determined from suitable fractographic results, if available. Recommended values of  $\sigma_z$  and guidelines for both straight bore and countersunk fastener holes are presented elsewhere [3,7].



## SECTION IV

### CONCLUSIONS AND RECOMMENDATIONS

#### 4.1 CONCLUSIONS

An advanced durability analysis (ADA) methodology has been developed for metallic aircraft structures. This methodology is very general and it can be used to evaluate durability design requirements for functional impairment due to: (1) excessive cracking, and (2) fuel leakage/ligament breakage. The ADA methodology accounts for the initial fatigue quality variation of structural details, the crack growth accumulation for a population of structural details under specified durability analysis conditions (e.g., material, load spectra, stress level, & bolt load transfer, etc.) and structural properties.

An "engineering approach", based on engineering principles, rather than mechanistic-based theories for microstructural behavior, is used to make the methodology useful for durability design applications. Hence, the ADA methodology is considered to be a "durability design tool." This perspective is very important for durability analysis applications.

The initial fatigue quality of a structural detail (e.g. fastener hole, cutout, fillet, lug, etc.) is represented by an equivalent initial flaw size distribution (EIFSD) function. The EIFSD is based on the back-extrapolation of fractographic results to time zero using a deterministic crack growth approach (DCGA). Analytical methods have been developed and verified for determining a suitable EIFSD for the durability analysis of clearance-fit fastener holes in metallic aircraft structures.

The ADA methodology reflects a probabilistic approach for predicting: (1) the probability of crack exceedance,

$p(i, \tau)$ , at any service time and/or (2) the cumulative distribution of TTCI,  $F_T(t)$ , at any crack size. Deterministic and stochastic crack growth methods have been developed for making  $p(i, \tau)$  and  $F_T(t)$  predictions for any crack size and service time. For example, the ADA methodology applies to the small crack size range associated with excessive cracking (e.g.,  $< 0.05$ " ) and to large through-the-thickness cracks (e.g.,  $0.50$ " -  $0.75$ " ) associated with fuel leakage/ligament breakage.

The ADA methodology presented in this report (Vol. I) is evaluated and verified in Volume II [3]. The following conclusions and understandings are based on the work of this report as well as Volume II.

1. Actual initial flaws in the bore of manufactured fastener holes in metallic aircraft structures usually consist of random scratches, burrs, microscopic imperfections, etc. Different forms, locations and combinations of flaws exist in each fastener hole. Such flaws, except for gross manufacturing defects, cannot be reliably detected and quantified by NDE for production aircraft structures. In reality, the actual initial flaws in fastener holes produced by manufacturing and assembly are not physical "cracks" in the usual sense associated with the linear elastic fracture mechanics approach. Whatever the source of fatigue cracking may be, a practical method for representing the reality of the as-manufactured condition is needed for durability analysis. This is taken care of by the equivalent initial flaw size concept described in the following.

2. An equivalent initial flaw (EIFS) is an artificial crack size which results in an actual crack size at an actual point in time when the initial flaw is grown forward. It is determined by back-extrapolating fractographic results. It has the following characteristics: (1) an EIFS is an arti-

ficial crack assumed to represent the initial fatigue quality of a structural detail in the as-manufactured condition whatever the source of fatigue cracking may be, (2) it has no direct relationship to actual initial flaws in fastener holes such as scratches, burrs, microdefects, etc., and it cannot be verified by NDI, (3) it has a universal crack shape in which the crack size is measured in the direction of crack propagation, (4) EIFSs are in a fracture mechanics format but they are not based on linear elastic fracture mechanics (LEFM) principles and are not subject to such laws or limitations such as the "short crack effect," (5) it depends on the fractographic data used, the fractographic crack size range used for the back-extrapolation and the crack growth rate model used, (6) it must be grown forward in a manner consistent with the basis for the EIFS, (7) EIFSs are not unique - a different set is obtained for each crack growth law used for the back-extrapolation, and (8) EIFSs are not necessarily comparable with minimum crack sizes obtained from fractographic results from tear-down inspections.

3. An EIFS is not strictly "generic" because it depends on the following: (1) crack growth rate model used to back-extrapolate fractographic results, (2) conditions reflected in the fractographic results (e.g., material, type fastener/hole/fit, load spectra, stress level, % bolt load transfer, etc.), (3) crack growth approach used (deterministic or stochastic), (4) fractographic crack size range used (i.e., AL-AU), and (5) goodness-of-fit criterion. However, the real issue is not whether the EIFSs or EIFSD is generic or not. The important question is: "Can an EIFSD, based on the fractographic results for one or more data sets, be used to make reasonable durability analysis predictions for a different set of conditions (e.g., similar material, same type of load spectra (e.g., fighter, bomber or transport), similar type fastener/hole/fit but different stress levels and/or % bolt load transfers)?" The answer to this question is "yes"! A

fractographic data pooling procedure has been developed and verified for determining suitable EIFSD for durability analysis.

4. An EIFS and/or EIFSD should be used in the proper context for durability analysis applications. EIFSs must be grown forward in the same manner as such EIFSs were defined in the first place. For example, an artificial initial flaw size or EIFS should not be indiscriminately grown forward to any desired crack size using a LEFM-based crack growth program without first considering the "basis" for the EIFS or EIFSD. If this principle is not strictly followed, durability analysis predictions will not be consistent with the EIFSD and the predictions may not be reasonable.

5. Initial flaws can be selected from the EIFSD for a given probability of crack exceedance (e.g., 1/10000 or 0.0001). Such initial flaws are artificial - not actual physical flaws that can be compared with the minimum crack size obtained from the fractographic evaluation of tear-down inspection results.

6. The ADA methodology includes a fracture mechanics framework. However, an empirical-based crack growth model is used in the small crack size region rather than linear elastic fracture mechanics (LEFM) principles. LEFM analytical crack growth methods are used only for crack sizes and stress intensity ranges where such principles apply. Therefore, existing LEFM methods do not have to be modified to account for the "short crack effect" or threshold stress intensity for crack propagation. Since an empirically-based crack growth model is used, mechanistic theories for microstructural behavior are not required. Since the ADA methodology is intended as a "durability design tool," practical considerations for achieving reasonable durability analysis results are stressed rather than mechanistic theories for microstruc-

tural behavior. This philosophy in no way diminishes the importance of continuing research in microstructural behavior. Such research is important to develop a fundamental understanding of microstructural behavior and appropriate mechanistic-based theories.

7. A step-by-step data pooling procedure, based on the combined least square sums approach (CLSSA), has been developed for estimating the EIFSD parameters using one or more fractographic data sets. A scaling method is provided for normalizing the fractographic results for multi-hole test specimen to an equivalent single hole basis. Therefore, fractographic results for different data sets can be "mixed and matched" to obtain the EIFSD parameters. Data pooling increases the size of the data base. Since EIFSD parameters are estimated in a "global sense," the resulting EIFSD can be justified for more general durability analysis applications. The EIFSD parameters can be optimized for a given fractographic data set(s) by minimizing the total standard error.

8. EIFSD parameters for the functions considered (i.e., Weibull compatible, lognormal compatible, lognormal and two-parameter Weibull) can be estimated using either a "TTCI fit" or "EIFS fit." It doesn't matter which fit is used to determine the EIFSD parameters because the same results are obtained anyway. However, from a philosophical standpoint, the "TTCI fit" may be preferred because TTCIs are physical quantities that can be obtained directly from the fractographic results. On the other hand, the EIFSs are artificial flaws derived from the TTCIs using a suitable crack size-time relationship. For every TTCI there is a corresponding EIFS.

9. Each fractographic data set should be screened and plotted before the data set is used to estimate the EIFSD parameters for a given EIFSD function. Screening is recommended to check the consistency of the fractographic results so

that questionable results can be censored out. There are no "hard and fast" rules for censoring but unexplained extreme values should be scrutinized.

10. This program has further advanced the understanding (e.g., meaning and limitations) of an EIFS and EIFSD for durability analysis applications. The durability analysis methodology for determining a suitable EIFSD, including optimization of the EIFSD parameters, has been advanced. Therefore, confidence in the EIFSD for durability analysis has been increased. Durability analysis methods have also been developed and verified for covering both the small and large crack size regions.

11. Methods have been developed for determining the EIFSD and initial flaw size for a selected probability of crack exceedance. Such methods could be used to standardize the way aerospace contractors define initial flaw size from fractographic results for tear-down inspections and other fatigue tests. A standard method for defining initial flaw size from fractographic results is needed so that comparable initial flaw sizes can be obtained by different aerospace contractors.

12. The DCGA is recommended for durability analyses in the small crack size region (e.g.,  $< 0.10''$ ) and the EIFSD should be defined using the DCGA. A fractographic crack size range of  $AL-AU = 0.01'' - 0.05''$  is recommended for defining the EIFSD. The reference crack size for TTCIs,  $x_1$ , should lie in the AL-AU range (i.e.,  $AL \leq x_1 \leq AU$ ). A different EIFSD could be specified for applications to different crack size ranges. However, a single EIFSD for all crack sizes is preferred. If a single EIFSD for any crack size is used, the service crack growth master curve (SCGMC) for  $p(i, \tau)$  and  $F_T(t)$  predictions should be approximated by at least two curve segments. This aspect will be discussed later. In the

small crack size region a single segment DCGA SCGMC can be used.

13. A two-segment approach for durability analysis is recommended to cover the small and large crack size region. The cumulative distribution of TTCI,  $F_T(t)$ , can be obtained for a convenient reference crack size in the AL-AU range, by growing the EIFSD forward using a single segment DCGA SCGMC. For example,  $x_1 = 0.05$ " would be a reasonable reference crack size to use for TTCIs for two reasons: (1) TTCIs for  $x_1 = 0.05$ " can be directly verified by fractographic results and (2) an 0.05" crack size in a fastener hole is a reasonable maximum crack size that can be cleaned up by reaming the hole to the next fastener size. The two segment approach is attractive because the total fatigue life can be divided into two parts, i.e., crack initiation and crack propagation. Crack initiation could be used for small cracks (e.g.,  $<0.05$ " ) and crack propagation could be used for cracks  $>0.05$ ". The DCGA and the stochastic crack growth approach (SCGA) can be used to evaluate functional impairment due to fuel leakage/ligament breakage. Reasonable durability analysis predictions were obtained in the large crack size region (e.g., 0.50" - 0.75" through-the-thickness cracks) using the two segment DCGA. However, based on results presented in Volume II [3], the two-segment DCGA-SCGA, in which the first segment (or crack initiation) is based on the DCGA and the second segment (crack propagation) is based on the SCGA, is recommended for durability analysis in the large crack size region.

14. A simple crack growth rate model ( $da(t)/dt = Qa(t)$ ) was used to develop crack size-time relationships between EIFSs and TTCIs and to transform TTCID functions into a EIFSD function and vice versa. This model has been found to be very reasonable for durability analysis applications. Other crack growth models could also be used to define EIFSs and/or

EIFSD functions. The crack growth model used should provide the proper crack size-time trends for a wide range of fractographic results. Also, the model parameters for different fractographic data sets should be determined in such a way that the parameters for different data sets are directly comparable and that they are on the same baseline. The compatible type EIFSD function (i.e., EIFSD function is compatible with the TTCID function) must be consistent with the crack growth model used. Furthermore, the durability analysis predictions in the small crack size region must be consistent with the basis for the EIFSD. These guidelines emphasize the generality and flexibility of the ADA methodology developed.

15. Compatible and non-compatible type EIFSD functions were considered under this program. The "compatible" EIFSD function is derived from the TTCID function using a suitable deterministic crack growth law. The EIFSD function can also be assumed with no consideration given to the TTCID function whether it is physically meaningful or not. This type of EIFSD function is referred to as "non-compatible". The typical non-compatible type EIFSD functions (e.g., two-parameter Weibull and lognormal) used will generally have an "open" upper tail which implies that infinite initial flaw sizes are possible for some probability of crack exceedance. On the other hand, a compatible type EIFSD function has a "closed upper end" which imposes an upper bound limit on the initial crack size. Compatible type EIFSD functions are recommended for durability analysis (e.g., Weibull compatible and the lognormal compatible EIFSD functions).

16. An EIFS upper bound limit,  $x_u$ , has to be specified for a compatible type EIFSD function. This limit should be established in conjunction with the other parameters in the EIFSD function (e.g.,  $\alpha$  and  $\phi$  for the Weibull compatible EIFSD function) and the economical repair limit for fastener holes. The EIFSD parameters should be optimized to minimize



the error in the  $F_T(t)$  and  $p(i,T)$  predictions in the areas of most interest. For example, for  $F_T(t)$  predictions the area of most interest is the tail of the TTCID where TTCIDs are the smallest. For  $p(i,T)$  predictions the upper tail of the crack size distribution where crack sizes are the largest is of most interest. It has been determined in Volume II [3] that different EIFSD parameter values are obtained for a given  $x_u$ . The accuracy of  $F_T(t)$  and  $p(i,T)$  predictions seem to be insensitive to significant changes in the  $x_u$  value. Consequently, reasonable predictions for  $F_T(t)$  and  $p(i,T)$  were obtained using different  $x_u$  values. For example, different values of  $\alpha$  and  $\phi$  can be obtained for the Weibull compatible EIFSD function for  $x_u = 0.01$ " and  $0.03$ ". In this case, one  $x_u$  value is three times larger than the other  $x_u$  value - yet, both  $x_u$ ,  $\alpha$  and  $\phi$  combinations can still provide comparable durability analysis results. An EIFS upper bound limit range of  $x_u = 0.03$ " -  $0.05$ " is considered reasonable for durability analysis. This  $x_u$  limit is based on NDI, considerations, the economical repair limit for fastener holes, and the initial flaw size for damage tolerance requirements. Ideally, the probability of exceeding the economical repair limit should be zero at time zero (i.e., as-manufactured state). However, if  $x_u$  is greater than  $0.05$ ", the probability of exceeding the economical repair limit will not be zero.

17. "User friendly" computer software has been developed for an IBM compatible PC for implementing the durability analysis methods developed under this program. This includes software for: (1) storing fractographic data on floppy disk, (2) screening and plotting fractographic results, (3) optimizing the EIFSD parameters for one or more fractographic data sets, (4) checking and plotting goodness-of-fit of  $F_T(t)$  and  $p(i,T)$  predictions based on given EIFSD, (5) making durability analysis predictions based on DCGA and/or SCGA. A software user's guide is available in Volume V [6].

## 4.2 RECOMMENDATIONS

The following recommendations are based on the work performed under this program:

1. The "durability analysis tools" need to be incorporated into the overall durability design process for metallic aircraft structures so that aerospace contractors can effectively use the tools developed.

2. The durability analysis methodology should be extended to interference fit and cold worked fastener holes and other structural details such as cutouts, lugs, fillets, etc. Appropriate test specimens should be designed and demonstrated for acquiring the fractographic data needed to define the EIFSD for other structural details. Guidelines and procedures are needed for applications.

3. Further experience is needed in developing service crack growth master curves, compatible with the basis for the EIFSD used, using an analytical crack growth program [e.g., 38,39]. Specifically, the analytical crack growth program used should be tuned or "curve fitted" to the EIFS master curves for selected fractographic data sets. The "tuned" analytical crack growth program should be used to make crack exceedance predictions for different durability design conditions. Then, the predictions should be verified using actual fractographic results. More experience is needed in setting up the crack growth analysis for durability applications, including the applicable assumptions that go into a damage tolerance analysis. Increased confidence in the durability analysis predictions for a wide range of design conditions is needed.

4. The ADA methodology should be used to investigate an existing aircraft structure. Durability design trade-offs

such as force structural maintenance requirements, risk of functional impairment due to excessive cracking and fuel leaks or ligament breakage, life cycle costs, aircraft readiness, structural weight and life should be quantified and the trade-offs evaluated and discussed.

5. The initial flaw concepts and methods developed under this program for durability analysis should also be investigated and evaluated for damage tolerance, risk analysis, and force structural maintenance applications.

6. Probability of crack detection aspects should be incorporated into the method for defining an EIFSD for structural details.

7. The effects of crack growth interactions on durability analysis predictions for  $p(i,T)$  and  $F_T(t)$  should be investigated - particularly for large cracks in adjacent fastener holes or other structural details. Durability analysis research so far has not addressed this issue. As long as the dominant fatigue crack in adjacent fastener holes is relatively small (e.g.,  $< 0.05"$ ), the growth of one fatigue crack should not significantly effect the growth of a neighboring crack and vice versa.

8. The effect of bolt load transfer on the EIFSD parameters and the service crack growth master curve need to be investigated further. Fatigue test results should be acquired using test specimens where the amount of bolt load transfer is controlled. The double-reverse dog bone specimens used under this program and the previous program had a variable amount of bolt load transfer due to the nature of the specimen design. Therefore, the effects of bolt load transfer on the EIFSD parameters, initial flaw size, and SCGMC could not be properly evaluated. The effect of the  $\frac{1}{2}$  bolt load transfer can be accounted for in the LEFM crack growth

program used. However, further work is needed to tune the SCGMC to the same basis as the EIFSD with the  $\frac{1}{2}$  bolt load transfer properly accounted for.

9. Fatigue test results, based on test specimens with a preflawed fastener hole, should be investigated for use in tuning the analytical crack growth program. More experience is needed in developing the SCGMC for specified design conditions in conjunction with the EIFSD.

10. Tear down inspection results from various aircraft should be used to determine the EIFSD for different types of materials and fastener hole types. EIFSDs should be determined from applicable fractographic results. Initial flaw sizes should be determined for selected crack exceedances. Then, the initial flaw sizes for different aircraft and materials should be compared. The initial flaw sizes should be developed using the standard procedures described in this report so that the initial flaw sizes will be compatible.

11. The ADA methodology should be extended to advanced composites applications.

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## DEFINITIONS

1. Combined Least Square Sums Approach (CLSSA) - the least square sums for individual fractographic data sets are combined to estimate the EIFSD parameters in a "global sense." This approach is used in conjunction with the data pooling philosophy.

2. Compatible Equivalent Initial Flaw Size Distribution Function - this is a distribution function for equivalent initial flaw sizes (EIFS) which is derived using a physically meaningful cumulative distribution of time-to-crack initiation (TTCI) function and a suitable deterministic crack growth law.

3. Crack Size - is the length of a crack in a structural detail in the direction of crack propagation.

4. Data Pooling - is a concept for estimating the EIFSD parameters for a given EIFSD function for one or more fractographic data sets in a "global sense." A data pooling procedure is used to justify the EIFSD for more general durability analysis applications.

5. Deterministic Crack Growth Approach (DCGA) - Crack growth parameters are treated as deterministic values resulting in a single value prediction for crack length.

6. Durability - is a quantitative measure of the airframe's resistance to fatigue cracking under specified service conditions. Structural durability is concerned with the prevention of functional impairments due to: (1) excessive cracking and (2) fuel leakage/ligament breakage. Excessive cracking is concerned with relatively small subcritical crack sizes (e.g.,  $< 0.05"$ ) which affect functional impairment, structural maintenance requirement and life-cycle-costs. Such cracks may not pose an immediate safety problem. However, if the structural details containing such cracks are not repaired, economical repairs cannot be made when these cracks exceed a limiting crack size. Functional impairment due to fuel leakage/ligament breakage is typically concerned with large through-the-thickness cracks (e.g.,  $0.50" - 0.75"$ ). Although such cracks are usually subcritical, they affect the residual strength and may require increased maintenance action.

7. Durability Analysis - is concerned with quantifying the extent of structural damage due to fatigue cracking for structural details (e.g., fastener hole, fillet, cutout, lug, etc.) as a function of service time. Results are used to ensure design compliance with Air Force's durability design requirements.

8. Economic Life - is that point in time when an aircraft structure's damage state due to fatigue, accidental damage and/or environmental deterioration reaches a point where operational readiness goals cannot be preserved by economically acceptable maintenance action.

9. Economic Life Criteria - are guidelines and formats for defining quantitative economic life requirements for aircraft structure to satisfy U. S. Air Force Durability design requirements. The economic life criterion provides the basis for analytically and experimentally ensuring design compliance of aircraft structure with durability design requirements. Two recommended formats for economic life criteria are:

- o probability of crack exceedance
- o cost ratio: repair cost/replacement cost

10. Economic Repair Limit - is the maximum damage size that can be economically repaired (e.g., repair 0.03" - 0.05" radial crack in fastener holes by reaming hole to next size).

11. Equivalent Initial Flaw Size (EIFS) - is an artificial crack size which results in an actual crack size at an actual point in time when the initial flaw is grown forward. It is determined by back-extrapolating fractographic results. It has the following characteristics: (1) an EIFS is an artificial crack assumed to represent the initial fatigue quality of a structural detail in the as-manufactured condition whatever the source of fatigue cracking may be, (2) no direct relationship to actual initial flaws in fastener holes such as scratches, burrs, microdefects, etc., and it cannot be verified by NDI, (3) a universal crack shape in which the crack size is measured in the direction of crack propagation, (4) it's in a fracture mechanics format but EIFSs are not based on linear elastic fracture mechanics (LEFM) principles and are not subject to laws or limitations, such as the "short crack effect" [e.g., 31-37], (5) it depends on the fractographic data, the fractographic crack size range for the back-extrapolation, and the crack growth rate model used, (6) it must be grown forward in a manner consistent with the basis for the EIFS, (7) EIFSs are not unique - a different set is obtained for each crack growth law used for the back-extrapolation, and (8) EIFSs are not necessarily comparable with minimum crack sizes obtained from fractographic results from tear-down inspections.

12. Equivalent Initial Flaw Size Distribution (EIFSD) - is used to represent the initial fatigue quality variation of a structural detail. An EIFS is a random variable, and the EIFSD statistically describes the EIFS population.

13. EIFS Master Curve - is a curve (e.g., equation, tabulation of  $a(t)$  vs.  $t$  or curve without prescribed functional form) used to determine the EIFS value at  $t=0$  corresponding to a given TCI value at a specified crack size. Such a curve is needed to determine the IFQ distribution from

the TTCI distribution. The EIFS master curve depends on several factors, such as the fractographic data base, the fractographic crack size range used, the functional form of the crack growth equation used in the curve fit, etc. (Ref. EIFS).

14. Extent of Damage - is a quantitative measure of structural durability at a given service time. For example, the number of structural details (e.g., fastener holes, cutouts, fillets, etc.) or percentage of details exceeding specified crack size limits. Crack length is the fundamental measure for structural damage. The predicted extent of damage is compared with the specified economic life criterion for ensuring design compliance with U. S. Air Force durability requirements.

15. Generic EIFS Distribution - An EIFS distribution is "generic" if it depends only on the material and manufacturing/fabrication processes. An EIFSD is not strictly "generic" because it is based on fractographic results which reflect given conditions (e.g., load spectra, stress level, & bolt load transfer, etc.). For durability analysis, an EIFSD is established using the fractographic results for one or more data sets, and the resulting EIFSD is justified for a different set of conditions.

16. Initial Fatigue Quality (IFQ) - characterizes the initial manufactured state of a structural detail or details with respect to initial flaws in a part, component, or airframe prior to service. Actual initial flaws in a fastener hole are typically random scratches, burrs, microscopic imperfections, etc. Such flaws are not cracks per se like those associated with linear elastic fracture mechanics. The IFQ is represented by an equivalent initial flaw size distribution (EIFSD).

17. Probability of Crack Exceedance ( $p(i, \tau)$ ) - refers to the probability of exceeding a specified crack size,  $x_i$ , at a given service time,  $\tau$ . It can be determined from the statistical distribution of crack sizes and can be used to quantify the extent of damage due to fatigue cracking in fastener holes, cutouts, fillets, lugs, etc.

18. Reference Crack Size ( $a_0$ ) - This is the specified crack size in a detail used to reference TTCI's. The IFQ distribution is based on a selected reference crack size.

19. Service Crack Growth Master Curve (SCGMC) - This curve is used to determine the EIFS,  $y_{1,i}(\tau)$ , corresponding to an exceedance crack size  $x_i$  at time  $\tau$ . The probability of crack exceedance,  $p(i, \tau)$ , can be determined from the EIFS cumulative distribution for a given  $y_{1,i}(\tau)$ . The SCGMC is defined for the applicable design variables (e.g., stress level, spectrum, etc.), and it can be determined using either

test data or an analytical crack growth program. All SCGMC's must be consistent with the corresponding EIFS master curve and the fractographic data base. The SCGMC must be consistent with the basis for the IFQ distribution.

20. Statistical Scaling - is used to normalize the fatigue cracking resistance of all structural details in a test specimen and fractographic data set(s) to an equivalent single detail population basis. Statistical scaling, minimizes the fractographic reading requirements because fractographic results are required for only the largest fatigue crack per test specimen.

21. Stochastic Crack Growth Approach (SCGA) - an approach which directly accounts for the crack growth rate dispersion in the durability analysis.

22. Structural Detail - is any element in a metallic structure susceptible to fatigue cracking (e.g., fastener hole, fillet, cutout, lug, etc.).

23. Time-To-Crack-Initiation (TTCI) - is the time or service hours required to initiate a specified (observable) fatigue crack size,  $a_0$ , in a structural detail (with no initial flaws intentionally introduced).

24. TTCI Lower Bound Limit ( $\epsilon$ ) - is a cutoff value for TTCI's reflected in the IFQ model. It varies for a given  $a_0$  and it depends on the EIFS upper bound limit,  $x_u$ , and the EIFS master curve. TTCIs for a given crack size,  $x_1$ , should be  $\geq \epsilon$ . This Weibull distribution parameter provides a basis for quantifying the EIFS distribution for different TTCI crack sizes on a common baseline.

25. Upper Bound EIFS Limit ( $x_u$ ) - defines the largest EIFS in the initial fatigue quality distribution. Constraints on  $x_u$  for fatigue holes: largest EIFS in data set  $\leq x_u$  (0.03"-0.05").

## ACRONYNS

|       |   |  |
|-------|---|--|
| ADA   | = | Advanced Durability Analysis               |
| CLSSA | = | Combined Least Square Sums Approach        |
| DADTA | = | Durability and Damage Tolerance Assessment |
| DCGA  | = | Deterministic Crack Growth Approach        |
| EIFS  | = | Equivalent Initial Flaw Size               |
| EIFSD | = | Equivalent Initial Flaw Size Distribution  |
| FHQ   | = | Fastener Hole Quality                      |
| HEIFS | = | Homogeneous EIFS                           |
| IFQ   | = | Initial Fatigue Quality                    |
| LEFM  | = | Linear Elastic Fracture Mechanics          |
| LT    | = | Load Transfer Through the Fastener         |
| MM    | = | Method of Moments                          |
| NDE   | = | Non Destructive Evaluation                 |
| NDI   | = | Non Destructive Inspection                 |
| NLT   | = | No Load Transfer Through the Fastener      |
| SCGA  | = | Stochastic Crack Growth Approach           |
| SSE   | = | Sum Squared Error                          |
| TSE   | = | Total Standard Error                       |
| TTCI  | = | Time-to-Crack Initiation                   |



# LIST OF SYMBOLS

|                        |   |
|------------------------|---|
| $a$                    | = Crack Size  |
| $a_0$                  | = Reference crack size for given TTCIs  |
| $a(0)$                 | = EIFS = Crack size at $t=0$  |
| $\bar{a}(0)$           | = Mean EIFS   |
| $a(t)$                 | = Crack size at any service time $t$  |
| $a(t), a(t_1), a(t_2)$ | = Crack size at time $t, t_1$ and $t_2$ , respectively  |
| $a(T)$                 | = Crack size at service time $T$  |
| $a(\tau)$              | = Crack size at any service time $\tau$   |
| $AL, AU$               | = Lower and upper bound fractographic crack size, respectively, used to define the EIFSD parameters. Also used in conjunction with the SCGMC to define crack size limits for the small crack size region. |
| $AU'$                  | = Upper bound crack size limit for the large crack size region  |
| $b, Q$                 | = Crack growth parameters in the equation $\frac{da(t)}{dt} = Q[a(t)]^b$ . Used in conjunction with the IFQ model.  |
| $b_1, Q_2$             | = Crack growth rate parameters in the equation $da/dt = Q(a)^{b_1}$ associated with the one-segment DCGA or 1st segment of the two segment approach.  |

|                    |   |
|--------------------|---|
| $b_2, Q_2$         | = Crack growth rate parameters in the equation $da/dt = Q_2(a)^{b_2}$ for segment two of the two-segment DCGA.  |
| $B$                | = $-\alpha \ln \phi$ or $\ln(a(0))$   |
| $c$                | = $b - 1$ ; Used in conjunction with the IFQ model when the crack growth law, $\frac{da(t)}{dt} = Q[a(t)]^b$ is used and $b > 1.0$ .                      |
| $c_i$              | = $b_i - 1$ ; Used in conjunction with the SCGMC when $\frac{da(t)}{dt} = Q_i[a(t)]^{b_i}$ is used.<br>The subscript "i" refers to the ith stress region. |
| $c_1$              | = $b_1 - 1$   |
| $c_2$              | = $b_2 - 1$   |
| $COV(\bar{x})$     | = Coefficient of Variation of $\bar{x}$   |
| $COV(\bar{y})$     | = Coefficient of Variation of $\bar{y}$   |
| $\frac{da(t)}{dt}$ | = Crack growth rate as a function of time   |
| $f_{a(0)}(x)$      | = EIFS probability density function = $\frac{dF_{a(0)}(x)}{dx}$   |
| $f_T(t)$           | = $\frac{dF_T(t)}{dt}$  |
| $f_X(u)$           | = Probability density function of $X$ .   |

- $F_{a(0)}(x)$ 
  - EIFS cumulative distribution function for an "equivalent single hole population."
  
- $F_{a_l(0)}(x)$ 
  - Cumulative distribution of EIFS based on the largest fatigue crack per test specimen.
  
- $F_{a_{l_i}(0)}(x_{ij})$ 
  - Subscripted notation used for  $F_{a_l(0)}(x)$  used in conjunction with data pooling, where:  $j$  denotes the  $j$ th crack in the  $i$ th data set.
  
- $F_a(t)(x)$ 
  - Cumulative distribution of crack size at any service time  $t$  based on an "equivalent single hole population" EIFSD.
  
- $F_{a_l}(t)(x)$ 
  - Cumulative distribution of crack size at any service time  $t$  based on the largest fatigue crack per test specimen basis for the EIFSD.
  
- $F_T(t)$ 
  - TTCI cumulative distribution function
  
- $F_{T_{l_i}}(t)$ 
  - Cumulative distribution of minimum TTCIs based on the largest fatigue crack per test specimen.
  
- $F_{T_{l_i}}(t_{ij})$ 
  - Subscripted notation used for  $F_{T_l}(t)$  used in conjunction with data pooling, where:  $j$  =  $j$ th TTCI value in the  $i$ th data set.

$$F_{T_l}(t) \Big|_{\alpha_0, \phi_0}$$

- Predicted cumulative distribution of minimum TTCI based on a selected  $x_u$  and initial values of  $\alpha_0$  and  $\phi_0$  for the Weibull compatible EIFSD function. Used in conjunction with non-linear procedure for estimating the EIFSD parameters.

$l$

- No. of fastener holes per test specimen in which only the largest fatigue crack per specimen is included in the fractographic data set. Used in conjunction with the "statistical scaling" technique.

$l_i$

- Same as  $l$  with the subscript " $i$ " referring to the  $i$ th fractographic data set.

$L(\tau), \bar{L}(\tau)$

- Total and average number of details, respectively, in the entire component having a crack size  $>x_1$  at any service time  $\tau$

LT

- Load transfer through the fastener

M

- Total number of fractographic data sets used to estimate the EIFSD parameters

$N_i$

- Number of TTCI or EIFS values for the  $i$ th data set used in conjunction with the combined least square sums approach; or total number of details in the  $i$ th stress region.

|                                |   |
|--------------------------------|---|
| $N(i, \tau), \bar{N}(i, \tau)$ | - Total and average number of details, respectively, having a crack size exceeding $x_1$ at any service time $\tau$   |
| $p(i, \tau)$                   | - Probability that a detail in the $i$ th stress region will have a crack size $> x_1$ at the service time $\tau$   |
| $Q$                            | - Crack growth rate parameter in $da(t)/dt = Qa(t)$ referred to as "pooled $Q$ " value when parameter represents the typical crack growth for a given fractographic data set.   |
| $Q_i$                          | - Crack growth rate parameter in $da(t)/dt = Q_i a(t)$ . In conjunction with fractographic data pooling, $Q_i$ denotes the crack growth rate parameter for the $i$ th data set in $M$ data sets to be used to estimate the EIFSD parameters. In conjunction with SCGA-based EIFSS, $Q_i$ is the crack growth rate parameter for the $i$ th fatigue crack in a data set. |
| $s(\bar{\tau})$                | - Standard deviation of $\bar{\tau}$  |
| $s(\bar{x})$                   | - Standard deviation of $\bar{x}$   |
| $t, t_1, t_2$                  | - Flight hours at $t, t_1, t_2$ , respectively.   |
| $t_1$                          | - Time to initiate crack size $x_1$   |
| $T, T_{TCI}$                   | - Time-to-crack-initiation  |

- $u$  = A particular value of  $X$ .
- $x$  = crack size
- $x_1$  = Crack size used for  $p(i, \tau)$  predictions
- $x_u$  = Upper bound limit for EIFS
- $X$  = Lognormal random variable with a median of 1.0.
- $x_{ij}$  =  $\ln \ln(x_u/x_{ij})$ ; EIFS Fit  
 =  $\ln[\ln(x_u/x_1) + Q_i t_{ij}]$ ; TTCI Fit  
 =  $\ln \ln(x_u/x_{ij}) - \ln Q_i$ ; EIFS Fit  
 =  $\ln [\ln(x_u/x_1) + Q_i t_{ij}] - \ln Q_i$ ; TTCI Fit  
 =  $l_i \exp(-l_i \delta_{ij}^{\alpha_0}) \delta_{ij}^{\alpha_0} \ln \delta_{ij}$
- $y_{1i}(\tau)$  = An EIFS in the EIFSD corresponding to a crack size  $x_1$  at time  $\tau$  in the  $i$ th stress region. Value determined using the SCGMC.
- $y_{ij}$  =  $\ln \left[ -(1/l_i) \ln F_{a_i(0)}(x_{ij}) \right]$  ; EIFS Fit  
 =  $\ln \left\{ -(1/l_i) \ln \left[ 1 - F_{T_{l_i}}(t_{ij}) \right] \right\}$  ; TTCI Fit  
 =  $\phi^{-1} \left\{ 1 - \left[ F_{a_i(0)}(x_{ij}) \right]^{1/l_i} \right\}$  ; EIFS Fit  
 =  $\phi^{-1} \left\{ 1 - \left[ 1 - F_{T_{l_i}}(t_{ij}) \right]^{1/l_i} \right\}$  ; TTCI Fit  
 =  $-\alpha_0 l_i \exp(-l_i \delta_{ij}^{\alpha_0}) [\ln(X_u/X_1) + Q_i t_{ij}]^{\alpha_0} (\phi_0)^{-\alpha_0-1}$

|                           |   |
|---------------------------|---|
| $z$                       | = Log X   |
| $z_{ij}$                  | = $j/(N_i + 1) - 1 + \exp(-\ell_i \delta_{ij}^{\alpha_0})$  |
| $\alpha, \beta, \epsilon$ | = Weibull distribution parameters for shape, scale, and lower bound TTCI, respectively.   |
| $\Gamma(\ )$              | = Gamma function  |
| $\xi, \gamma$             | = Empirical constants in the equation:<br>$Q_i = \xi \sigma^\gamma$ , where $\sigma$ = stress   |
| $\sigma$                  | = Stress or standard deviation  |
| $\sigma_z$                | = Standard deviation of $da(t)/dt$ with respect to plot of $\ln da(t)/dt = \ln a(t) + Qt$   |
| $\tau$                    | = A particular service time   |
| $\alpha_k$                | = Initial $\alpha$ value for the kth data set used in conjunction with data pooling and Method of Moments   |
| $\alpha_0, \beta_0$       | = Two-parameter Weibull distribution shape and scale parameters for EIFS, $a(0)$ , respectively.  |
| $\alpha, \phi$            | = Weibull compatible shape and scale EIFSD parameters, respectively   |
| $\alpha_0, \phi_0$        | = Initial values of $\alpha$ and $\phi$ used in conjunction with the non-linear approach for estimating EIFSD parameters for the Weibull compatible EIFSD function. |

$\Delta\alpha_0, \Delta\phi_0$ = Increments of  $\alpha_0$  and  $\phi_0$ , respectively $\mu, \sigma$ = Mean value and standard deviation of  $\ln a(0)$  (or  $\ln x$ ), respectively $\Phi( )$ 

= Standardized normal distribution function

 $\lambda_{ij}$ 

$$= \ell_i^{1/\alpha_k} \ln(x_u/x_j)$$

 $\psi_{ij}$ 

$$= \ell_i^{1/\alpha_k} Q_i \tau_j$$

 $\gamma$ 

$$= Q_1/Q_2$$

 $\gamma_{ij}$ 

$$= \ln \left[ \frac{\ln(x_u/x_j) + Q_i}{\phi} \right]$$

 $\Lambda$ 

$$= [1 - (Q_1/Q_2)] \ln a_0; \quad b_1 = b_2 = 1$$

$$= \ln a_0 + Q_1 (c_2 Q_2)^{-1} (a_0)^{-c_2}; \quad b_1 = 1; \quad b_2 \neq 1$$

$$= (c_1 Q_1)^{-1} (a_0)^{-c_1} - (c_2 Q_2)^{-1} (a_0)^{-c_2}; \quad b_1 \neq 1; \quad b_2 \neq 1$$



## APPENDIX A

### DURABILITY ANALYSIS CONCEPTS, ISSUES, AND PHILOSOPHY

#### A.1 INTRODUCTION

Durability analysis concepts, key issues and philosophy are reviewed and discussed in this section. The time-to-crack-initiation (TTCI) and equivalent initial flaw size (EIFS) concepts are reviewed in terms of initial fatigue quality representation. Key issues are clarified and the probability of crack exceedance concept is discussed in terms of deterministic crack growth.

Initial fatigue quality (IFQ), the cornerstone of any durability analysis, is represented by an equivalent initial flaw size distribution (EIFSD). An "engineering approach" is used to represent the IFQ of the structural details to be included in the durability analysis. Although engineering principles, instead of mechanistic-based theories, are used to model the microcracking behavior, LEFM principles can still be used to perform the durability analysis. The nature of the approach, implications and perspective for durability analysis need to be reviewed and discussed. Various aspects are discussed based on the advancements and understandings of this program and past research. A conceptual approach is used and mathematical details are provided in other sections of this report (Vol. I).

#### A.2 TTCI CONCEPT

The time to initiate a natural fatigue crack in a structural detail to any crack size,  $x_1$ , is referred to as the "time-to-crack-initiation" (TTCI). Initial fatigue

quality of structural details can be represented using either a distribution of TTCI or equivalent initial flaw size (EIFS) [2,12,13,18]. TTCIs can be obtained from fractographic results, either directly, by interpolation or by extrapolation. For example, in Fig. A.1 the TTCI for each of the solid circles is equal to the corresponding time required to initiate the specific crack size.

### A.3 EIFS CONCEPTS

Actual initial flaws in the bore of manufactured fastener holes in metallic aircraft structures usually consist of random scratches, burrs, microscopic imperfections, etc. Under service conditions fatigue cracks may eventually initiate from such flaws and then propagate to crack sizes that can be detected by NDI. However, the actual flaws in fastener holes produced by manufacturing and assembly are not typical "cracks" in the usual sense associated with the LEFM approach. Whatever type of initial flaw may be in a fastener hole, such flaws must be accounted for to assure that the structure will be durable and can be economically maintained. Some means is needed to quantify the initial fatigue quality of actual initial flaws in such a way that their effect on structural durability can be accounted for.

An equivalent initial flaw is an artificial crack with a characteristic size measured in the direction of crack propagation. Such a flaw is not an actual physical flaw. EIFSs are determined by back-extrapolating fractographic results to time zero (Fig. A.1). The initial fatigue quality is represented by an equivalent initial flaw size distribution (EIFSD). This is an "engineering approach" to the durability analysis problem. The objective of such an approach is to provide a practical "design tool" that can be used to evaluate Air Force durability design requirements and tradeoffs for metallic aircraft structures.

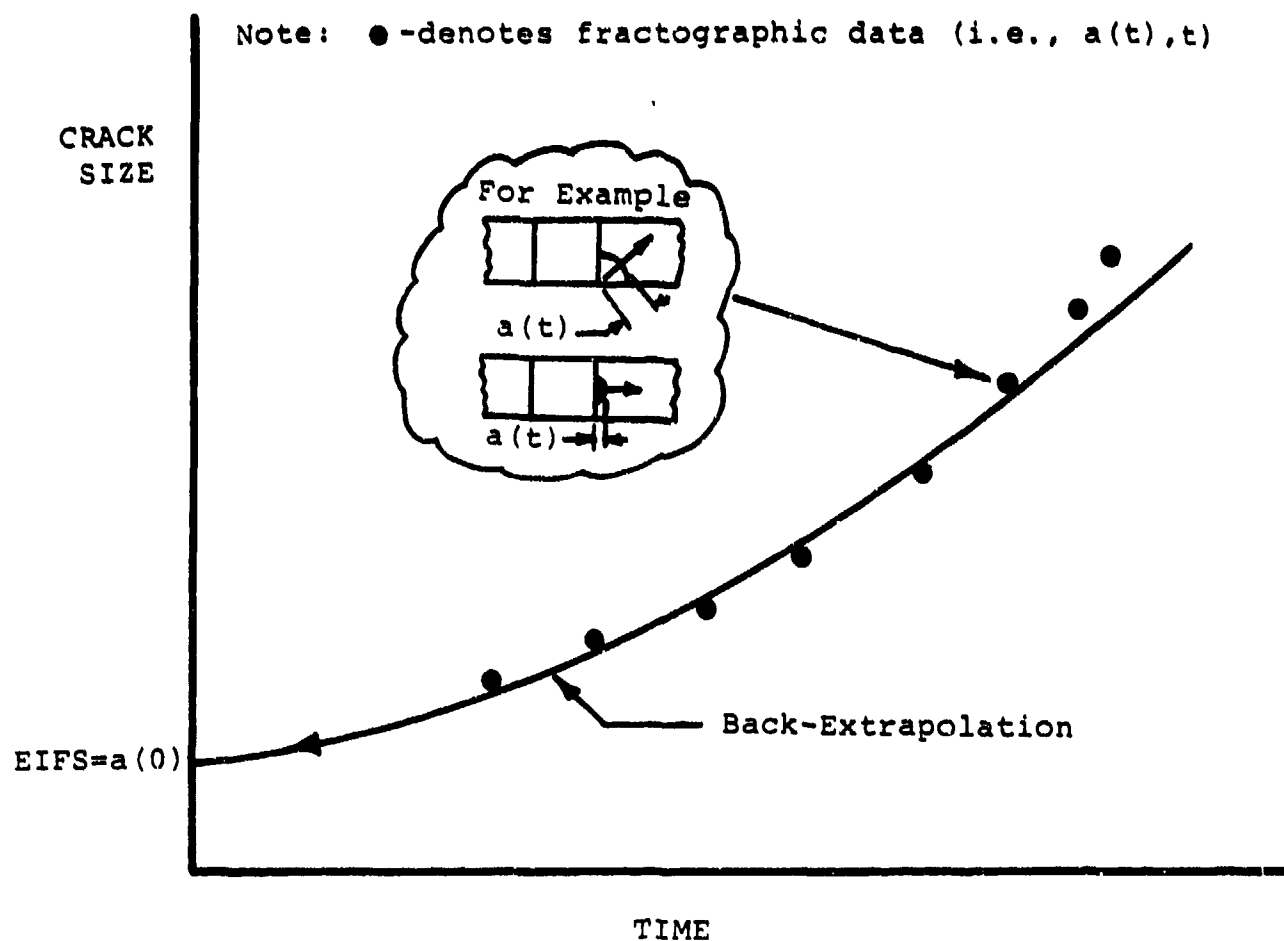


Figure A.1. An Equivalent Initial Flaw is Determined by Back-extrapolation of Fractographic Results.

EIFSs are generally defined using either a deterministic crack growth approach (DCGA) or a stochastic crack growth approach (SCGA). The two approaches are conceptually described in Fig. A.2 and further details are given in Appendix E.

In Fig. A.2, suppose that EIFSs are to be determined for both the DCGA and the SCGA using the same fractographic data. Using the DCGA, the EIFS for each fatigue crack is determined using the same crack size-time relationship or "EIFS master curve." Each EIFS has a corresponding time-to-crack-initiation at a given reference crack size and vice versa. As illustrated in Fig. A.2(a), the EIFSs and TTCIs have the same rank in the respective populations. With the DCGA, the variance in the crack growth rate, is directly reflected in the EIFS value computed.

Using the SCGA, the EIFS for each fatigue crack is determined using a crack size-time relationship that is separately fitted to each fatigue crack. In other words, each fatigue crack is back-extrapolated to time  $t = 0$  using a crack size-time relationship that is tailored for each fatigue crack. Whereas, the DCGA uses a single EIFS master curve to determine the EIFSs, the SCGA uses a different EIFS curve to obtain the corresponding EIFS for each fatigue crack. The SCGA is conceptually described in Fig. A.2(b). Unlike the DCGA, the crack growth trajectories for the SCGA may cross over each other as depicted in Fig. A.2(b). Since each fatigue crack is back-extrapolated independently, the crack growth rate dispersion is filtered out of the EIFS computed.

The "basis" for an EIFS and/or EIFSD fracture includes consideration of such factors as: (1) fractographic crack size range, AL-AU, used, (2) material, (3) fatigue testing conditions (e.g., spectrum, stress level, %, bolt load



transfer, etc.), (4) manufacturing process, and (5) crack growth law and procedure for back-extrapolation, including goodness-of-fit criterion. An EIFS and/or EIFSD should be used in the proper context for durability analysis. For example, an artificial initial flaw size or EIFS should not be indiscriminately grown forward using a LEFM-based crack growth program without considering the "basis" for the EIFS or EIFSD. If this principle is not strictly followed, durability analysis predictions will not be consistent with the IFQ, and the results will be questionable.

An EIFS is a random variable. Each structural detail to be used in the durability analysis is assumed to have a single dominant flaw which can be represented by an equivalent initial flaw size (EIFS). The population of EIFSs is represented by an equivalent initial flaw size distribution (EIFSD) (Fig. A.3). Initial flaw sizes can be selected from the EIFSD which correspond to a given probability of crack exceedance. However, such initial flaws are artificial - not actual physical flaws. For example, an equivalent initial flaw from an EIFD, for a given probability of crack exceedance (e.g., 1/10000 or 0.0001), has no direct relationship to the minimum crack size obtained from the fractographic evaluation of tear-down inspection results (Fig. A.4).

The EIFSD reflects a fracture mechanics format. However, if the EIFSD is based on an empirical crack growth law, the growth of the EIFSD forward must be consistent with the crack growth law used. A LEFM crack growth program can be used to grow the EIFSD forward but only in terms of the applicable crack growth law reflected in the EIFSD. An artificial initial flaw should not be grown forward using a LEFM program without considering the basis for the EIFS. (Fig. A.5).

\* Measured in Direction of Crack Propagation

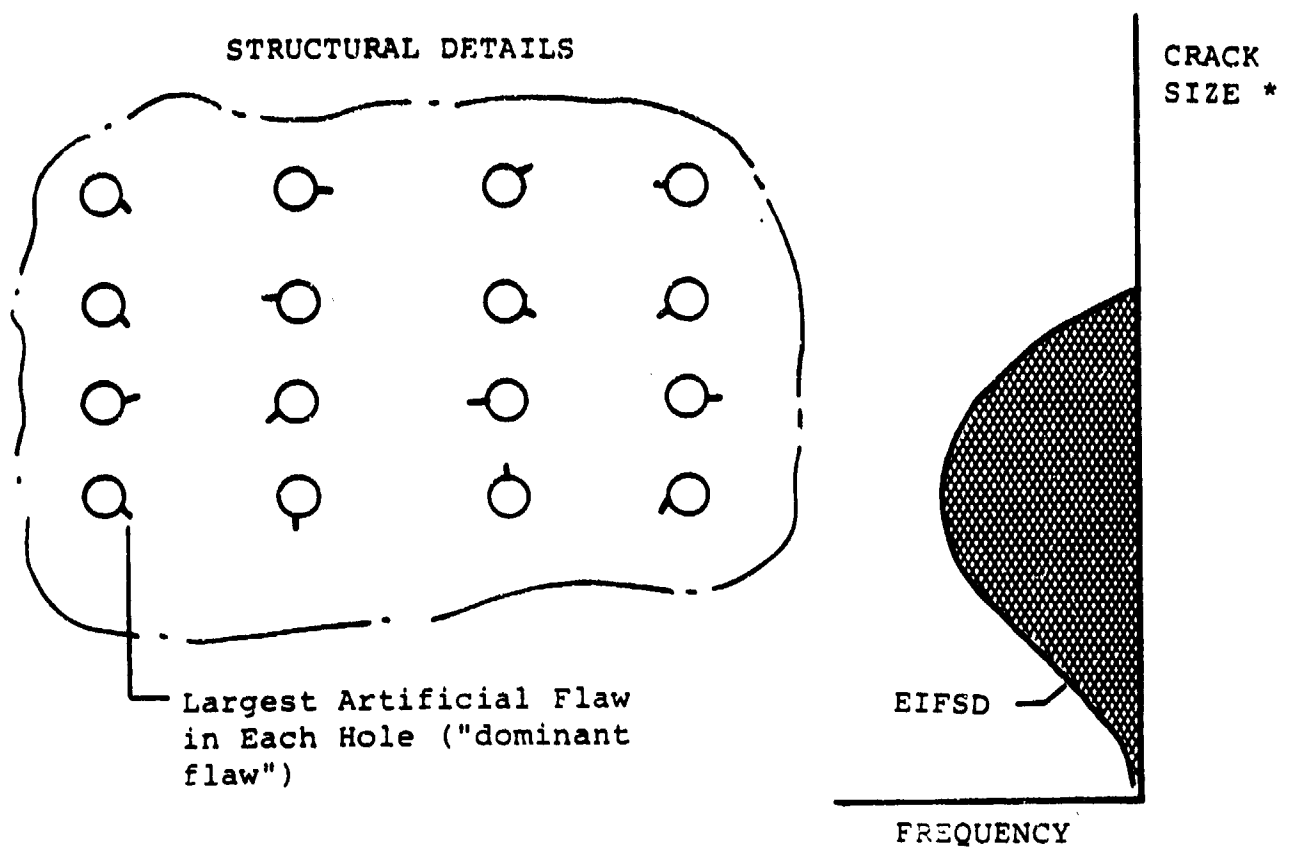
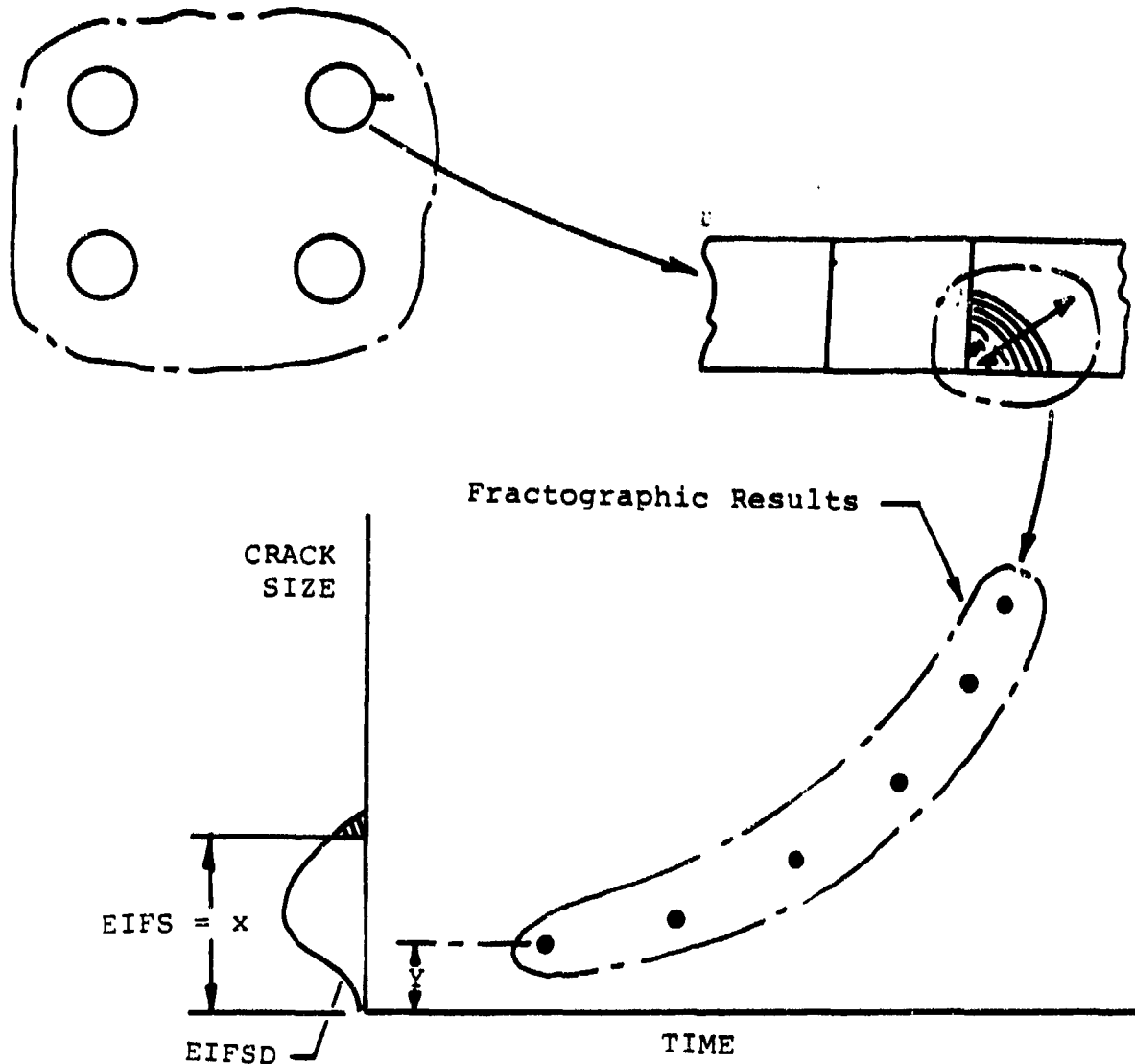


Figure A.3. An Equivalent Initial Flaw Size Distribution Represents the Initial Fatigue Quality of Structural Details.

# TEAR-DOWN INSPECTION



Y = minimum crack size from fractographic evaluation

Figure A.4. An Artificial Initial Flaw Size is Not Necessarily Comparable to the Minimum Crack Size Associated with Tear-Down Inspection Fractographic Results.



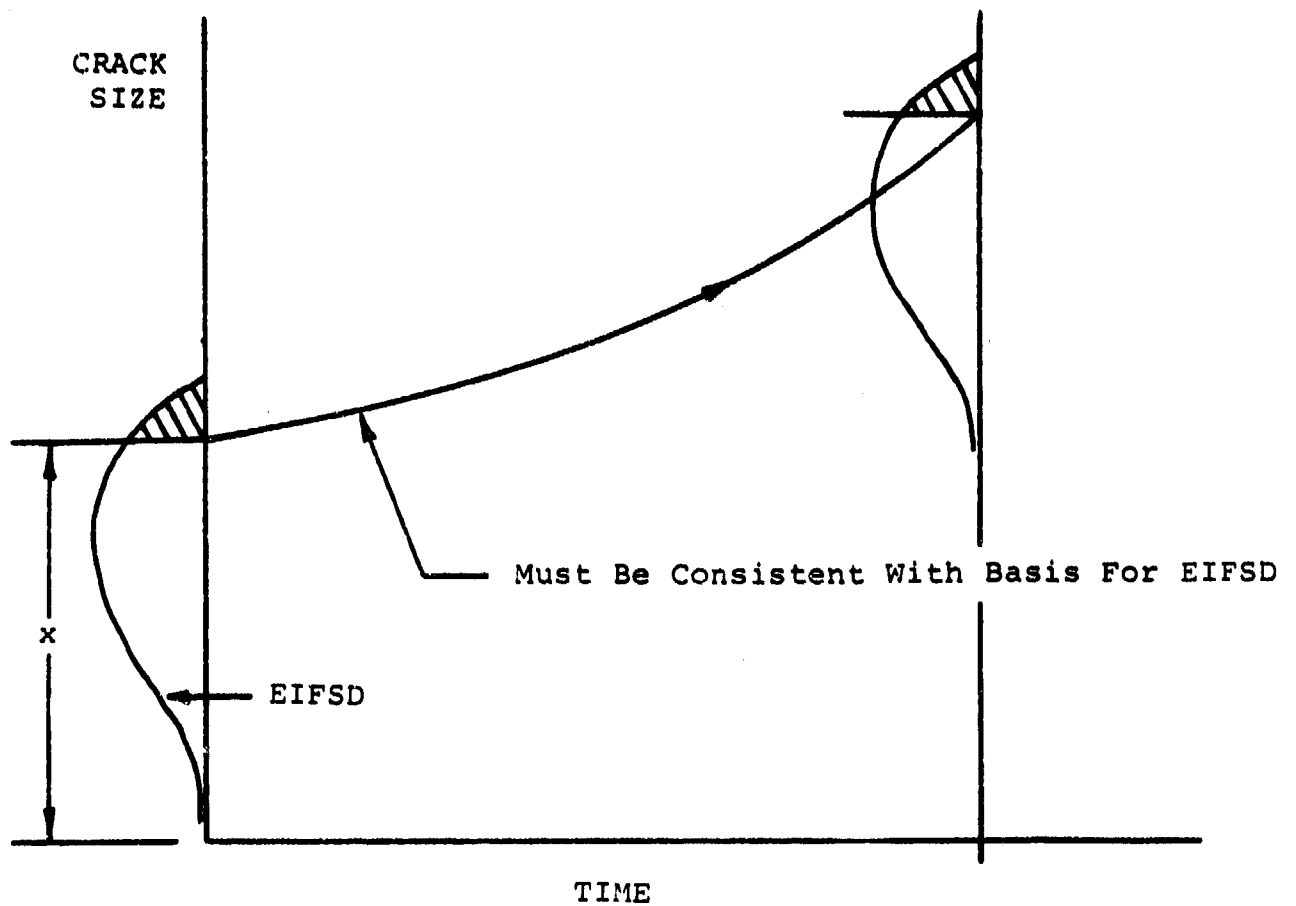
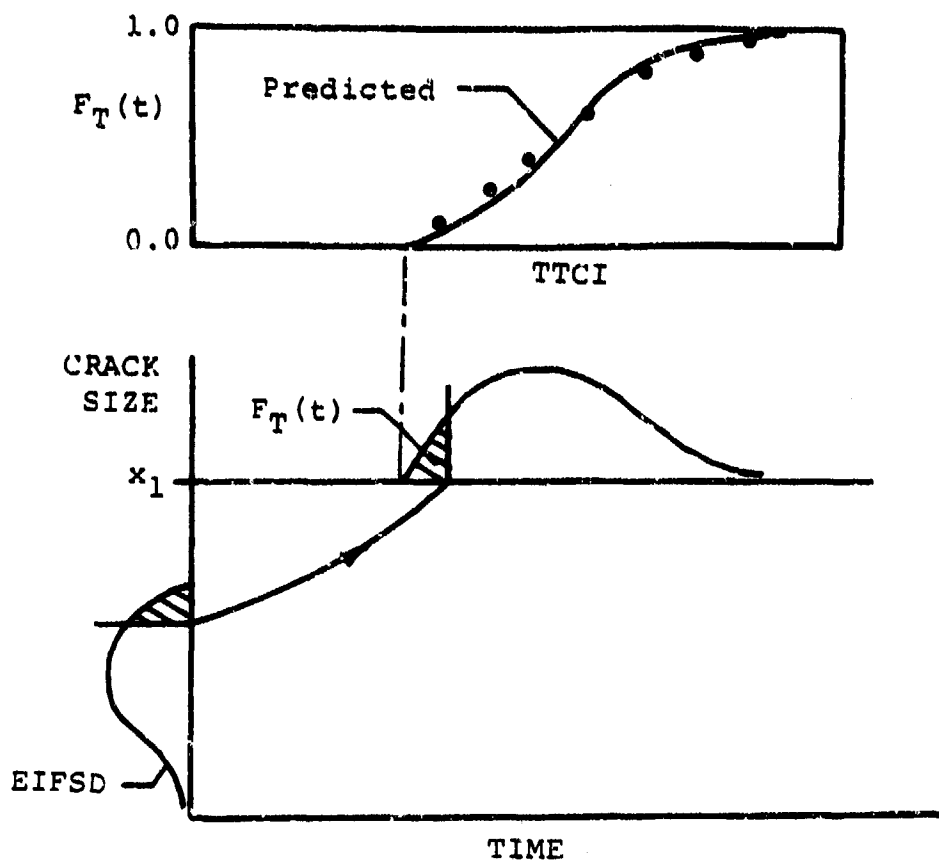


Figure A.5. An EIFSD or Initial Flaw Size Must be Grown Forward Consistent with the Basis for the EIFSD.

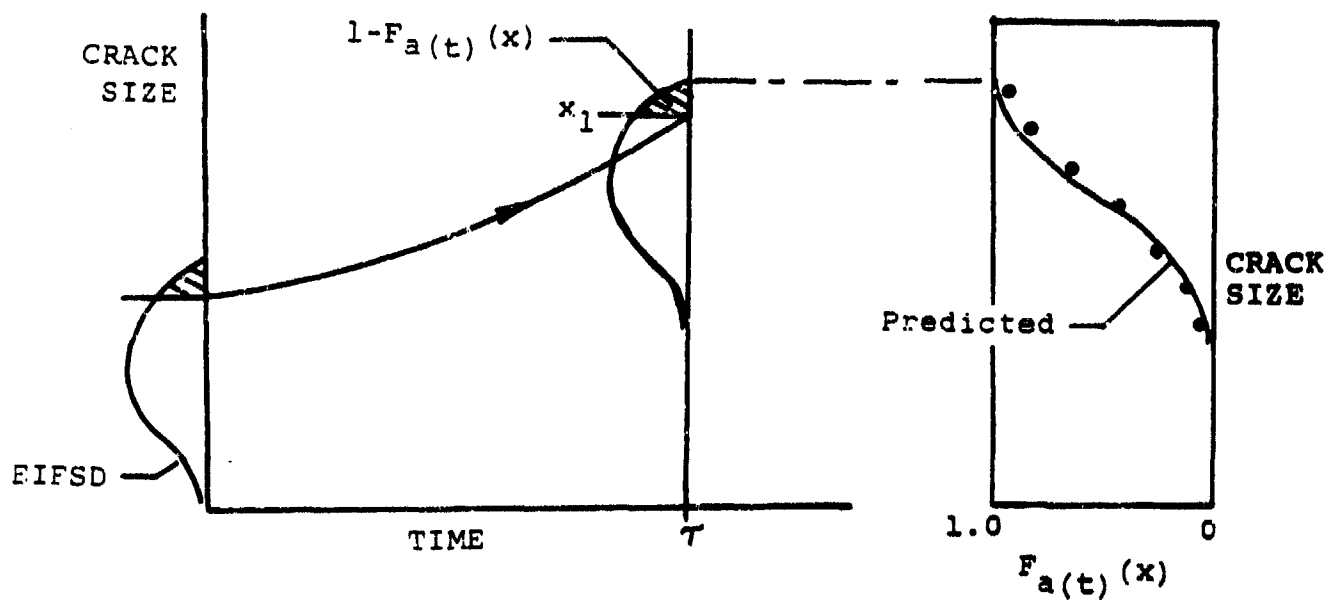
Since an equivalent initial flaw is not an actual physical flaw which can be verified by NDE, then how can an EIFSD be justified for durability analysis? An EIFSD can be justified by showing that reasonable predictions for the cumulative distribution of crack or cumulative distribution of TTCI can be obtained using the given EIFSD. Such predictions can be correlated with actual fractographic results for given conditions (e.g., load spectrum, stress level, & bolt load transfer, etc.) (Fig. A.6).

The concept of a single dominant flaw in a fastener hole(s) is used. (Fig. A.7). This concept greatly simplifies the durability analysis and is an important aspect of the overall method.

Fatigue cracks originate in fastener holes. Multiple micro-cracks with different geometries and locations may occur in fastener holes. During microcrack initiation and crack growth, there can be multiple branching with complex interactions between the various cracks affecting the crack growth rates and damage accumulation. In general, the micro-cracks eventually merge into a single dominant crack. The crack growth rates for the dominant crack can be determined from the fractographic results (Fig. A.7). Each fastener hole or holes sharing a common fastener have a single dominant crack in the fastener hole (Fig. A.7). The effects of the various cracks in the fastener hole on the crack growth rate is accounted for in the experimental data. Therefore, the effects of microcracking and crack interactions on the crack growth rate and damage accumulation are accounted for. The concept of a single dominant flaw in a fastener hole greatly simplifies the analysis. Each fastener hole is assumed to have a single dominant flaw, and the crack growth of the dominant flaw in each fastener hole is assumed to be independent of the crack growth of the dominant flaws in surrounding fastener holes.



(a) Correlate  $F_T(t)$  Prediction with Test Results



(b) Correlate  $F_a(t)(x)$  Prediction with Test Results

Figure A.6. Correlate Predictions Based on EIFSD with Test Results.

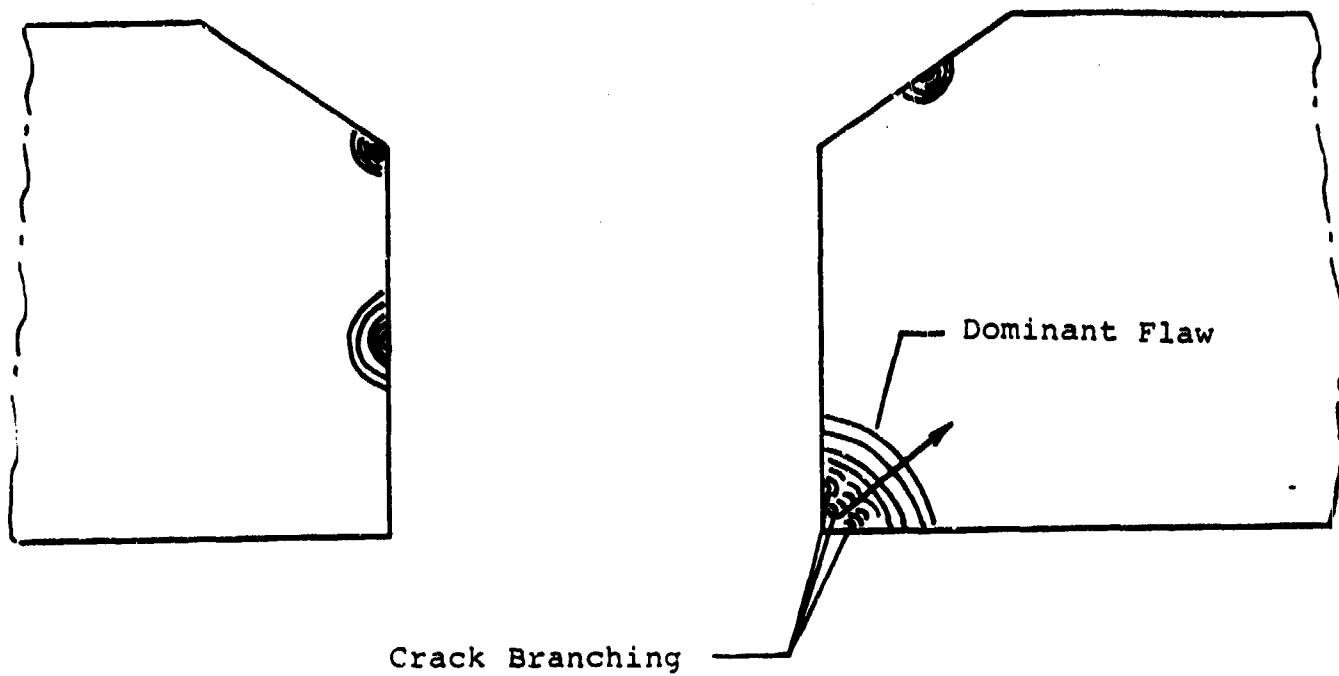


Figure A.7. Fatigue Cracks in Fastener Holes May Have Different Initiation Sites and Exhibit Crack Branching.

#### A.4 CLARIFICATION OF EIFS ISSUES

The following issues have been raised about the EIFS approach for durability analysis: (1) the EIFS approach does not account for limitations of LEFM and the "short crack effect," (2) a threshold stress intensity may exist below which cracks do not propagate (3) an EIFS and/or EIFSD is not a "generic" property of the material and manufacturing processes (4) EIFS does not necessarily account for crack geometry and shape. These issues are discussed and clarified in this section.

Before addressing the above issues, we will clarify the use of an EIFSD for representing the initial fatigue quality of a structural detail. There are two basic durability design requirements: (1) excessive cracking and (2) fuel leakage/ligament breakage. The excessive cracking requirements are concerned with relatively small fatigue cracks (e.g.,  $<0.05$ "). For example, to predict the probability of exceeding an  $0.05$ " crack size in a fastener hole at any time,  $t$ , a reasonable estimate of the initial flaw size distribution and a means for growing the population of initial flaws forward is needed.

Ideally, we would like to define the geometry, shape and location of actual initial flaws in the bore of a fastener hole due to material and manufacturing processes using conventional NDI methods. Such initial flaws are too small to detect and accurately quantify in a physical sense by direct NDI means. If actual initial flaws could be accurately quantified (i.e., location, geometry, shape) in fastener holes, two major problems would still have to be resolved: (1) how to accurately characterize actual initial flaws mathematically and (2) how to predict crack initiation by such flaws. Actual initial flaws in the bore of a fastener hole can have

different shapes, geometry and location. Moreover, multiple micro-flaws may exist, and such flaws may have different crack initiation sites under service conditions. Furthermore, during the initial stage of the crack initiation, growth and interaction of such cracks is very complex. For example, microcrack branching can occur in which the growth of one crack affects the growth to another crack and vice versa. With multiple crack initiation sites and crack branching, cracks tend to merge into a common crack front (Fig. A.7).

A suitable mechanistic model for characterizing the crack initiation process in the small crack size region is simply not available to implement the probabilistic-based durability analysis approach. Therefore, an "engineering approach" is used to define the initial fatigue quality. The "engineering approach" is considered to be a "design-tool" for evaluating airframe durability design requirement and tradeoffs. The goal of the "engineering approach" is to provide reasonable durability analysis results. Research work should be continued to develop a better understanding of the crack initiation process and an effective model. In the meantime, an "engineering approach" is recommended for durability analysis.

#### A.4.1 LEFM Limitations and "Short Crack Effect"

It is well known that linear elastic fracture mechanics (LEFM) laws break down for small crack sizes. Although the minimum crack size for valid LEFM applications is debatable, a minimum size of 0.005" [59] is considered reasonable. The crack growth rate is typically faster for "short cracks" than for "long cracks" for the same stress intensity range [e.g., 31-37].

The EIFS approach and philosophy for durability analysis in view of the above considerations will now be explained in a conceptual manner using Fig. A.8. Three different approaches are illustrated in Fig. A.8 for determining the EIFS value using actual fractographic results in a selected AL-AU crack size range. In each of the three cases the fractographic results are extrapolated backwards to time zero to obtain an EIFS value. An empirical-based crack growth law is assumed for case 1. In case 2, it is assumed that the LEFM program is simply a "curve fitting tool" applicable to any crack size. In this case the "short crack effect" is ignored entirely. In case 3, the back-extrapolation procedure accounts for both the "short crack effect" and limits the use of LEFM to crack sizes where most consider LEFM methods to be valid. It is assumed in case 3 that the mechanistic-based model is used to accurately characterize the crack initiation process in the small crack size region.

In Fig. A.8 it is assumed that the empirical crack growth law is fitted on a least squares fit basis to the fractographic results in the AL-AU range. For cases 2 and 3, it is assumed that a trial and error curve fitting method is used and the goodness-of-fit is based on a subjective decision by the analyst. The resulting EIFS value for cases 1, 2, and 3 is denoted in Fig. A.8 by  $x_1$ ,  $x_2$ , and  $x_3$ , respectively. Observations and interpretation of the results depicted in Fig. A.8 are: (1) a different EIFS value is obtained for each case considered, (2) using the fractographic results for a given AL-AU range, the same EIFS value for case 1 should be obtained by any analyst if the least squares fit criterion and the same fitting procedures are used, and (3) since the goodness-of-fit is subjective for cases 2 and 3, no two analysts will necessarily get the same initial flaw sizes.

\* Considered as "curve fitting tool" to justify in "short crack" region

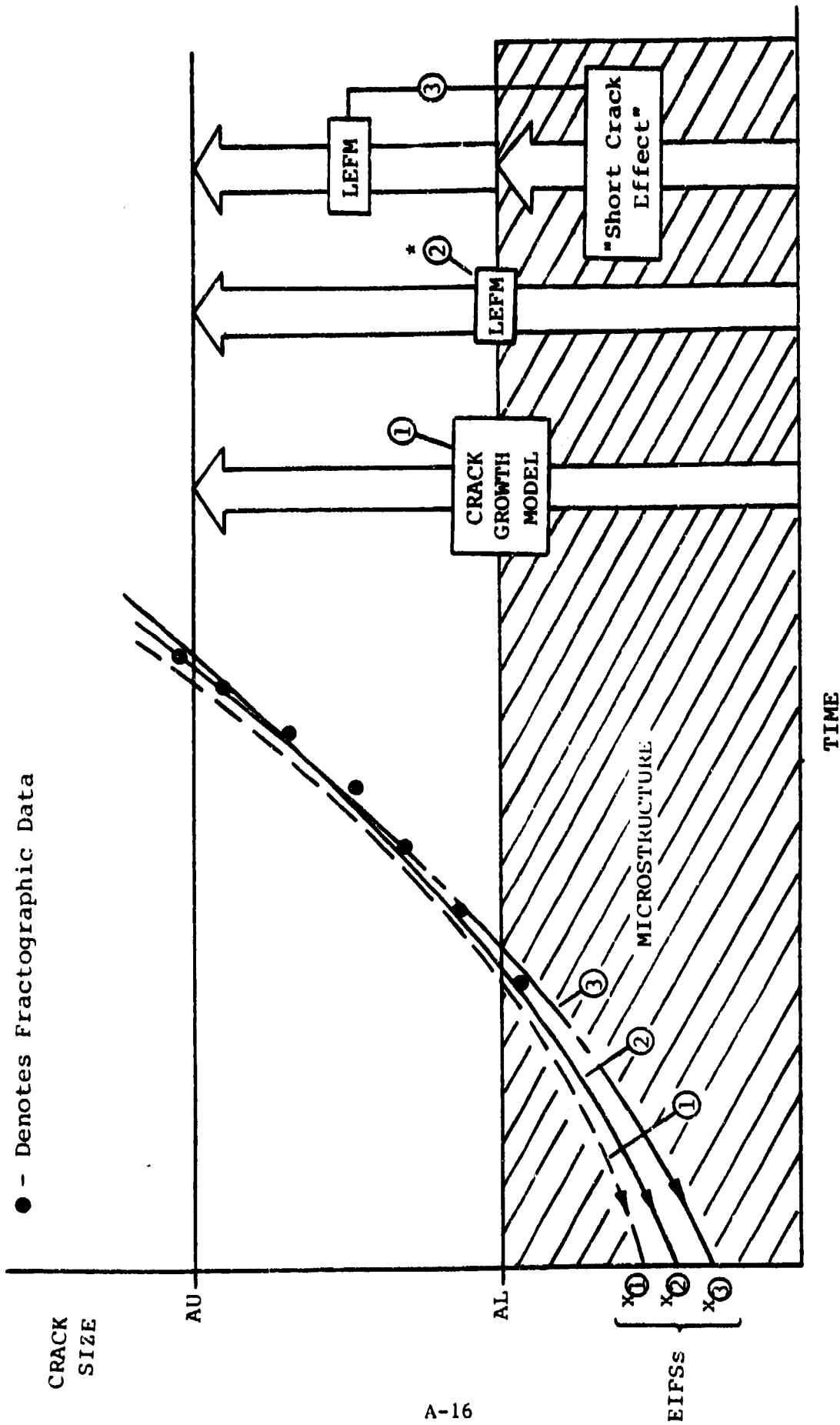


Figure A.8. Equivalent Initial Flaw Size (EIFS) Depends on Approach Used to Back-extrapolate Fractographic Results In A Selected AL-AU Range.



No standard analytical crack growth program is used by most aerospace companies today. Most companies usually develop their own working versions for fracture mechanics applications. Some companies may try to incorporate the "short crack effect" into their program while others do not. Different analytical crack growth programs are in use today.

The back-extrapolation procedure initially developed for determining EIFSs for the F-4 DADTA program [57] is based on an analytical crack growth program and a subjective decision by the analyst is required about goodness-of-fit. In view of the different versions of analytical crack growth that are currently being used and the subjective decision factor, it's not surprising that EIFSs for the same material and manufacturing process differ so much from company-to-company. There has been a real need for some time now to develop a universal procedure for defining EIFSs and EIFSDs so that the same initial fatigue quality results will be obtained by any analyst or aerospace company for the same fractographic data set.

#### A.4.2 Consistent EIFSs and Crack Growth Predictions

In any durability analysis, it is very important that EIFSs be defined in a consistent manner. Furthermore, the EIFSs must be grown forward in the same manner that the EIFSs were determined. If this principle is not strictly followed inconsistent EIFSs and durability analysis results will be obtained. To conceptually illustrate this crucial point consider that the EIFS value  $x_2$  shown in Fig. A.8 will be grown forward using each of the three approaches indicated. In Fig. A.9, crack growth predictions are shown for each of the three approaches based on an initial flaw size of  $x_2$ . The

\* Considered as "curve fitting tool" to justify in "short crack" region

● - Denotes Fractographic Data

$x_2$  = EIFS Based on Approach ②

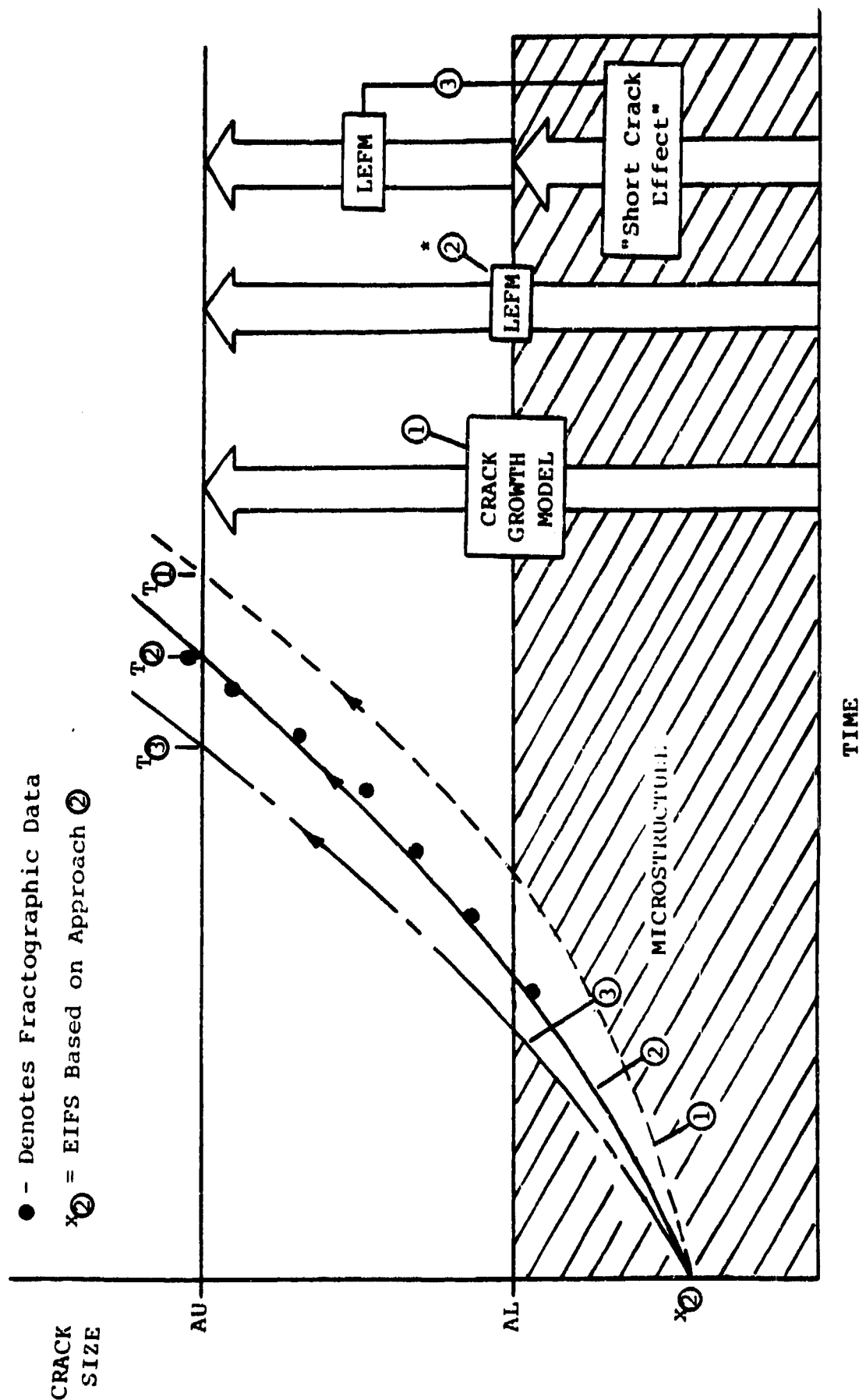


Figure A.9. Crack Growth Predictions Must Be Consistent with the Basis of the Equivalent Initial Flaw Size.

actual fractographic data in the AL-AU crack size range used in the back-extrapolation procedure, is denoted by a solid circle (●). The crack growth prediction shown in Fig. A.9 reflects the same EIFS master curve shown in Fig. A.8.

The following observations are made about the results depicted in Fig. A.9: (1)  $x_2$  was defined based on approach 2 and since  $x_2$  was grown forward the same way the fractographic results were back-extrapolated the predictions are consistent with the basis of the EIFS (note: in this case the EIFS master curve and the SCGMC are identical), (2) if the EIFS  $x_2$  is grown forward using the SCGMCs for either approach ① or ③, the crack growth predictions are not consistent. For example, the crack growth predictions for approaches ① and ③ are unconservative and conservative, respectively, compared to the observed fractographic results.

#### A.4.3 Generic EIFS and EIFSD Considerations/Philosophy

Ideally, EIFSs and/or EIFSDs should be "generic quantities" that depend only on the material, actual initial flaws (e.g., scratches, voids, imperfections), and manufacturing processes. Moreover, such initial flaws should have the same geometry, shape and details as actual physical flaws that may be present in as-manufactured fastener holes. Unfortunately, most as-manufactured initial flaws cannot be economically detected or quantified using state-of-the-art NDE techniques. If actual initial flaws could be quantified, other problems would still remain: (1) how to mathematically describe and adequately characterize actual initial flaws and (2) how to model the crack initiation process in a mechanistic sense.

The crack initiation behavior in the microstructure is very complex. Such factors as plasticity, persistent slip bands, grain boundaries, crack branching, interactions between microcracks, etc. are important considerations for de-

veloping a physically meaningful model for crack initiation. Research is continuing in this important technical area. A suitable mechanistic-based crack initiation model is not currently available for practical durability design and analysis applications. In lieu of such a model, an "engineering approach" is used for durability analysis.

Are EIFSS and/or EIFSDs generic? Since EIFSS are determined by back-extrapolating fractographic results, they are not strictly "generic", because the fractographic results are obtained from fatigue cracks generated under specified test conditions (e.g., load spectrum, stress level, % bolt load transfer, fastener hole type/fit, etc.). Therefore, the EIFSS depend on the test conditions reflected. This implies that EIFSS and/or EIFSDs have to be developed from fractographic data reflecting the applicable conditions not only for the EIFSS but also for the crack growth predictions. Since there are many possible combinations of durability analysis variables (i.e., load spectrum, stress level, % bolt load transfer, fastener hole type/fit), this could result in a prohibitively expensive acquisition program.

The real issue of the EIFS approach is not whether the EIFSS or EIFSDs are "generic" or not but rather can reasonable durability analysis predictions be obtained for a given set of conditions using EIFSS based on another set of conditions. A data pooling procedure has been developed for quantifying EIFSD parameters for one or more fractographic data sets. Different conditions can be reflected in the definition of the EIFSD. This provides a way for using the resulting EIFSD for variables outside the range of the baseline conditions. The derived EIFSD, based on the data pooling procedure, is justified for more general durability analysis by showing that predictions for the distribution of TTCI at a given crack size or the distribution of crack size at a given

time correlate reasonably well with the observed fractographic data base used to define the EIFSD. The analyst must understand the philosophy and justification for using the EIFS approach.

#### A.4.4 Shape and Geometry of EIFSs

EIFSs are determined by back-extrapolating fractographic results for fastener holes. The shape and geometry as well as crack initiation sites may vary depending on the type of fastener hole/fit as well as the amount of load transfer through the fastener. For example, it has been observed that the dominant fatigue crack in an unloaded fastener hole typically originates in the bore of a hole [e.g., 16, 58]. Whereas, the dominant fatigue crack in a bolt hole, where the bolt is loaded in single shear, usually occurs at the bolt hole interface with the mating part [e.g., 16, 58]. For crack growth analysis, the fatigue crack in the bore of the hole might be represented by a through-the-thickness initial flaw. In a similar manner, the fatigue crack at the bolt hole interface might be represented by an initial corner flaw. The shape of the flaw and the number of flaws in a fastener hole affects the stress intensity factor. For example, the stress intensity factor for a single corner flaw is different than that for two corner flaws on opposite sites of a hole or for a single through-the-thickness flaw. Decisions about flaw shape, size and locations are common considerations in damage tolerance analysis. Such decisions may vary depending on the analyst. This simply points out that judgments frequently have to be made about the input to the LEFM crack analysis. Similar judgments and assumptions have to be made to implement the probabilistic-based durability analysis methodology.

The "crack size" reflected in the fractographic data used in the EIFS determination is typically measured in the

direction of crack propagation. In this context, the shape of the crack is not directly considered in the determination of EIFSs or EIFSDs. An equivalent initial flaw is assumed to be a "characteristic crack size" which applies to different flaw shapes in a crack growth analysis. Using the data pooling procedure, EIFSs for different fractographic results and possibly different actual flaw shapes are treated as a single population of EIFSs for purposes of determining the EIFSD.

#### A.4.5 Initial Flaw Sizes for Fuel Leakage/ Ligament Breakage Requirements

One of the objectives of this program was to develop a method for defining an initial flaw size for a given crack exceedance probability. In the beginning we hoped that a single initial flaw size could be used to make crack growth predictions for any crack size associated with fuel leakage/ligament breakage requirements. The purpose of this section is to discuss the determination of initial flaw size and understandings reached under this program, regarding meaning and use of an EIFS.

Three key issues are: (1) how to determine an EIFS for fuel leakage/ligament breakage applications for a given crack exceedance probability and confidence level, (2) what limitations, if any, are placed on the growth of the EIFS to various crack sizes, and (3) how to interpret EIFSs in terms of actual in-service cracks. An EIFS for a given probability of crack exceedance can be defined directly from the EIFSD. The size of the equivalent initial flaw depends on: (1) the fractographic crack size range, AL-AU, used to define the EIFSD, (2) the approach used to back-extrapolate the fractographic results (e.g., empirical crack growth law or fracture mechanics-based analytical crack growth program), (3) crack growth approach (deterministic or stochastic), (4) goodness-of-fit criterion, (5) statistical distribution function used

to define the EIFSD, (6) EIFS upper bound limit ( $x_u$ ), and (7) desired confidence level. The following fundamental principle must be strictly followed when crack growth predictions are made using an EIFS: crack growth predictions must be consistent with the basis for the EIFS used. For example, if the EIFSD is defined for a selected fractographic crack size range (e.g., AL-AU = 0.01" - 0.05") and given empirical crack growth law (e.g.,  $da(t)/dt = Qa(t)$ ), the EIFS used for crack growth predictions is also limited to applications using the same crack size range and empirical crack growth law.

Either of the two following approaches can be used to define an EIFS from the EIFSD and to grow the EIFS forward to crack sizes associated with fuel leakage/ligament breakage (e.g., 0.50" - 0.75"): (1) use a single EIFSD and two or more crack growth segments to make crack size predictions in the desired crack size range and (2) define the EIFSD based on the fractographic results for the crack size range where crack growth predictions are described; then define the service crack growth master curve considering the same AL-AU range used to define the EIFSD. The two approaches above are conceptually described and compared in Fig. A.10.

Any initial flaw size based on an EIFD must be considered in the same context as the EIFSD was determined. This means that the EIFS should be interpreted considering everything involved in its definition (e.g., fractographic data used, AL-AU crack size range, crack growth law used for back-extrapolations, goodness-of-fit criterion, etc.). EIFSs are not physical cracks per se and should not be treated as such. Equivalent initial flaws are artificial cracks whose characteristics size has no direct relationship to actual minimum crack sizes associated with fractographic results from tear-down inspections or other fatigue test results. This perspective is extremely important to assure that initial flaws





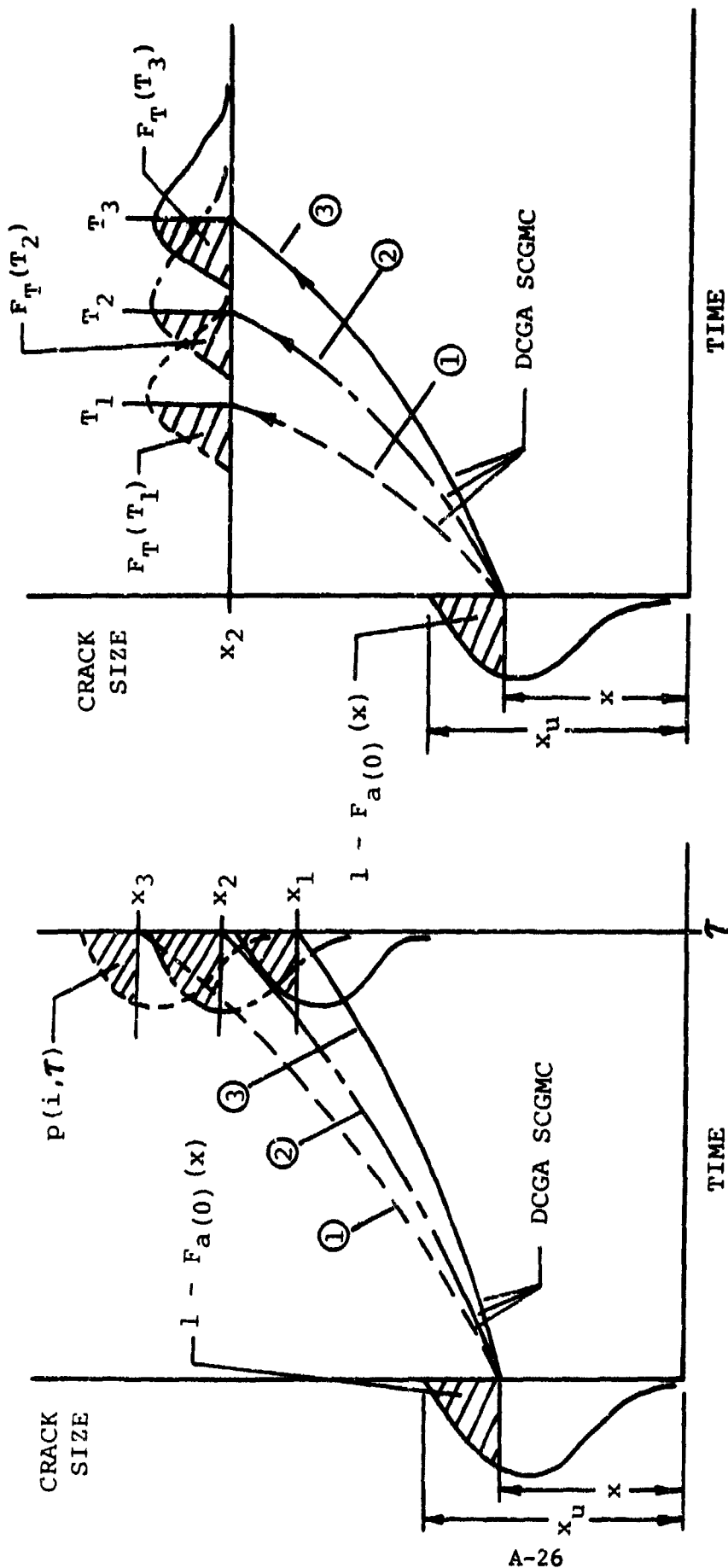
based on an EIFSD are properly used in any durability analysis.

#### A.5 PROBABILITY OF CRACK EXCEEDANCE/ CUMULATIVE DISTRIBUTION OF TTCI

The two most important predictions in any durability analysis are: (1) probability of crack exceedance at any time  $t$ ,  $p(i, \tau)$ , and (2) cumulative distribution of TTCI for any reference crack size  $x_1$ ,  $F_T(t)$ . Assume the EIFSD has been defined and that a deterministic crack growth approach is used to grow the EIFSD forward using a service crack growth master curve (SCGMC) which reflects selected design variables. For example, in Fig. A.11 predictions for  $p(i, T)$  and  $F_T(t)$  are illustrated for three different stress levels, where  $\sigma_1 > \sigma_2 > \sigma_3$ . All SCGMCs must be consistent with the basis for the EIFSD. In Fig. A.11(a), the probability of exceeding crack size  $x_1$ ,  $x_2$  and  $x_3$  is equal to the cross-hatched area under the crack size distributions shown. Therefore, for equal  $p(i, \tau)$   $x_1 > x_2 > x_3$ .

Predictions for the cumulative distribution of TTCI,  $F_T(t)$ , are illustrated in Fig. A.11(b). In this case, the cross-hatched area under the TTCID represents  $F_T(t)$ . Also note in Fig. A.11(b) that for the same  $F_T(t)$  value,  $T_1 < T_2 < T_3$ . This trend would be expected since  $\sigma_1 > \sigma_2 > \sigma_3$ .

SCGMCs can be determined using either suitable fractographic results or a LEFM analytical crack growth program [e.g., 38, 39]. Details and guidelines for determining SCGMCs which are compatible with the basis for the EIFSD are given in Sections II and III.



(a) Probability of Crack Exceedance Predictions,  $p(i, \tau)$ , at Time

(b) Cumulative Distribution of TTCI Predictions,  $F_T(t)$ , at Selected Reference Crack Size

Figure A.11. Concept of Predicting the Probability of Crack Exceedance and/or Cumulative Distribution of TTCI Using an EIFSD and SCGMC.

## APPENDIX B

### STATISTICAL SCALING TECHNIQUE

The EIFS distribution,  $F_{a(0)}(x)$ , for fastener holes is defined for a "single hole population." Test specimens for acquiring fatigue crack growth data may have one or more fastener holes per specimen. Some specimens may not be fatigue tested to failure. Also, every fastener hole in each replicate test specimen may not contain a measurable fatigue crack or else the crack is too small or complex (o.g., multiple crack origins and branching) for fractographic analysis. Hence, a statistical scaling technique has been developed for determining the equivalent initial flaw size distribution for a single hole population using the largest fatigue crack per specimen [2, 13]. This minimize the fractographic reading requirements, permits a maximum utilization of the available fractographic data and allows for "mixing and matching" of fractographic data for specimens with a different number of holes.

A technique is described in this section for statistically scaling the TTCID or EIFSD. This general technique, conceptually described in Fig. B.1 applies to any distribution function used to represent the TTCID or EIFSD. Various aspects are discussed.

Although every fastener hole in replicate test specimen may be equally stressed, there's no guarantee that every hole will have a measurable fatigue crack when the specimen fails or when the fatigue test is stopped before failure. In any case, the cracking resistance of each fastener hole in each IFQ test specimen should be accounted for when determining the EIFSD.

# Notes

- ① Distribution of TTCI minimums based on largest fatigue crack in 1 of  $l$  fastener holes/specimen
- ② TTCID for "single hole population"
- ③ Distribution of EIFS maximums based on the largest fatigue crack in 1 of  $l$  fastener holes/specimen
- ④ EIFSD for "single hole population"

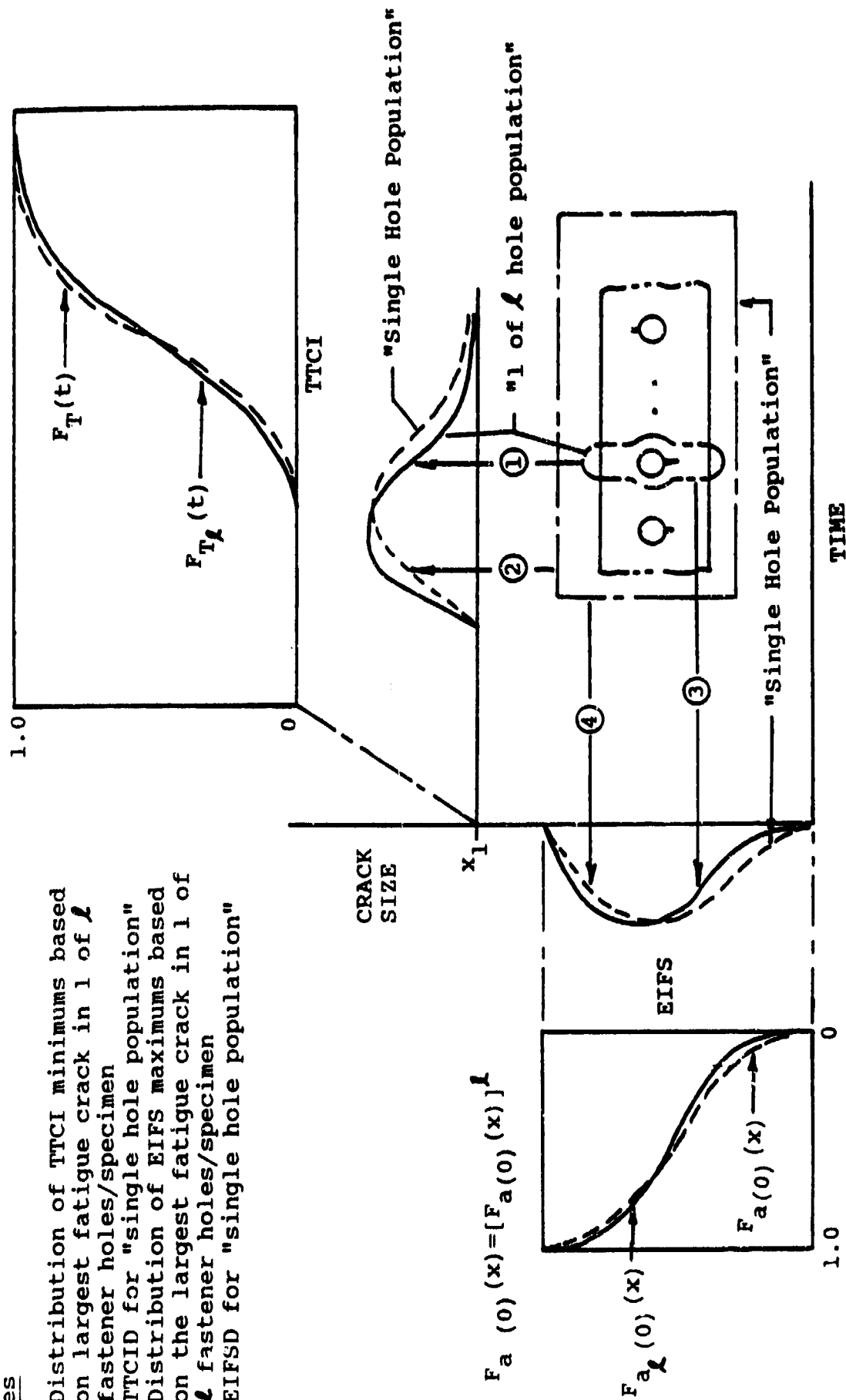


Figure B.1. Statistical Scaling Concept for "Single Hole Population".

There's another important aspect about scaling. With scaling, the EIFSD can be determined using only the largest fatigue crack per specimen. Test specimens for different replicate data sets may contain one or more fastener holes per specimen. For example, the test specimens in one replicate data set may have only one fastener hole per specimen and those in another data set may contain four fastener holes per specimen. Scaling accounts for the number of fastener holes in each specimen and is the common denominator for obtaining homogeneous results for different fractographic data sets.

A "single hole population" accounts for the actual number of fastener holes/specimen in the fractographic data base used to define either the TTCID or the EIFSD. There are two ways to do this: (1) use the fractographic data for the largest fatigue crack in each fastener hole/specimen, or (2) use only the largest fatigue crack in 1 of  $\ell$  holes per specimen and a scaling technique to account for 1 of  $\ell$  holes per specimen (Fig. B.1).

Let the cumulative distribution of EIFS for a single hole population be denoted by  $F_{a(0)}(x)$  and that of the maximum EIFS, based on the largest fatigue crack per specimen with fastener holes, be denoted by  $F_{a(0)}(x)$ . Assuming that fatigue cracking in each equally stressed fastener hole of a specimen is statistically independent of that of the other holes,  $F_{a(0)}(x)$  is related to  $F_{a(0)}(x)$  as shown in Eq. B-1,

$$F_{a_{\ell}(0)}(x) = \left[ F_{a(0)}(x) \right]^{\ell} \quad (B-1)$$

where  $\ell$  = number of equally stressed fastener holes per specimen.

Let the cumulative distribution of crack size at any time  $\tau$  for a "single hole population" be denoted by  $F_a(\tau)(x)$  and that for a "1 of  $\ell$  hole population" be denoted by  $F_{a\ell}(\tau)(x)$ .  $F_a(\tau)(x)$  and  $F_{a\ell}(\tau)(x)$  can be determined from  $F_{a(0)}(x)$  and  $F_{a\ell(0)}(x)$ , respectively, using a service crack growth master curve, (SCGMC) denoted by  $y_{11}(\tau)$ . Similarly, let  $F_T(\tau)$  and  $F_{T\ell}(\tau)$  denote the cumulative distribution of TTCI for a "single hole population" and for a "1 of  $\ell$  hole population", respectively. The notations and meanings are illustrated in Fig. B.2.

The cumulative distribution of crack size for any time,  $\tau$ , can be determined from the EIFSD using the applicable SCGMC. When the initial flaw size,  $y_{11}(\tau)$ , is grown forward using the SCGMC, a crack size  $x_1$  is obtained at time,  $\tau$ , (Fig. B.3).

The cumulative distribution of crack size at any time,  $\tau$ , is obtained from the respective EIFSDs as follows. In Eqs. B-2 and B-3,  $y_{11}(\tau)$  is the initial flaw size corresponding to a crack size  $x_1$  at time,  $\tau$ , as determined from the SCGMC. An expression for  $y_{11}(\tau)$  is given in Eq. B-4.

$$F_{a(\tau)}(x) = F_{a(0)} \left[ x = Y_{11}(\tau) \right] \quad (B-2)$$

$$\begin{aligned} F_{a\ell(\tau)}(x) &= F_{a\ell(0)} \left[ x = Y_{11}(\tau) \right] \\ &= \left\{ F_{a(0)} \left[ x = Y_{11}(\tau) \right] \right\}^\ell = \left\{ F_{a(\tau)}(x) \right\}^\ell \end{aligned} \quad (B-3)$$

$$Y_{11}(\ell) = x_1 \exp(-Q_1 \tau) \quad (B-4)$$

The cumulative distribution of crack size is assumed to be statistically compatible with the cumulative distribution of TTCI. Equations B-5 through B-8 reflect this assumption.

$$F_T(\ell) = 1 - F_{a(\tau)}(x) \quad (B-5)$$

- 

CRACK SIZE

TIME

①

②

TTCID

SCGMC

$F_{T_i}(t)$

$F_T(t)$

$1 - F_{a(0)}(x)$

$1 - F_{a_x(0)}(x)$

$x_1$

$T$

$Y_{1,i}(T)$

Figure B.2, Notations and Concepts for Cumulative Distribution of Crack Size and TTCL.

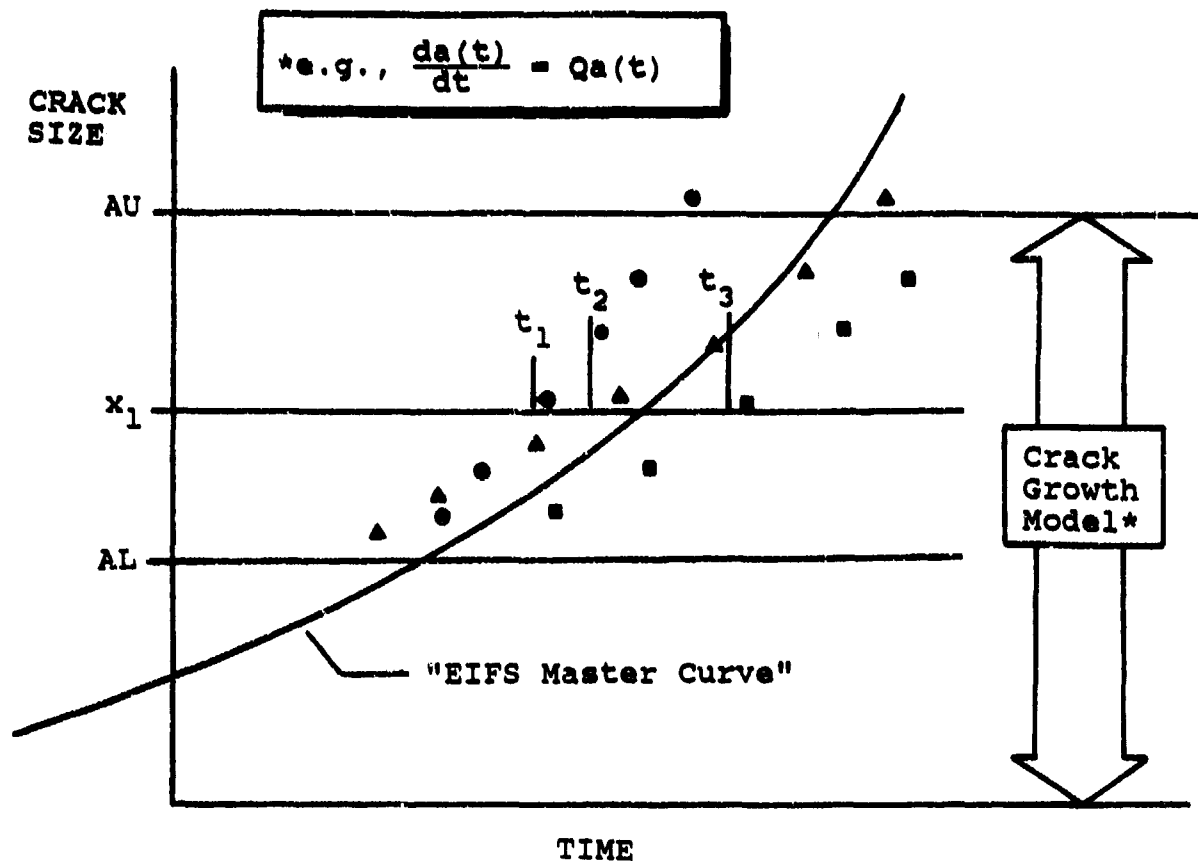


Figure B.3 Determine EIFS Master Curve Using Fractographic Data in Selected AL-AU Range and Suitable Crack Growth Model.



$$F_{a_i}(x) = 1 - F_T(t) \quad (B-6)$$

$$F_{T_\ell}(t) = 1 - F_{a_\ell}(x) \quad (B-7)$$

$$F_{a_\ell}(x) = 1 - F_{T_\ell}(t) \quad (B-8)$$

If the largest fatigue crack in each fastener hole is statistically independent of that of the other holes,  $F_T(t)$  and  $F_{T_\ell}(t)$  are related as follows in Eqs. B-9 and B-10.

$$F_T(t) = 1 - \left[ 1 - F_{T_\ell}(t) \right]^{1/\ell} \quad (B-9)$$

$$F_{T_\ell}(t) = 1 - \left[ 1 - F_T(t) \right]^\ell \quad (B-10)$$

Equations B-1, B-9 and B-10 are the key relationships for scaling. These equations are useful when the data pooling procedure is used to estimate the EIFSD parameters for an equivalent single hole population.

In some cases, fractographic results may not be available for each fastener hole per specimen in a replicate data set. Whenever scaling is used to define the EIFSD, it must also be accounted for when the EIFSD is grown forward to check the goodness-of-fit of the cumulative distribution of crack size and/or TTCI.

## APPENDIX C

### DERIVATION OF EQUATIONS FOR DETERMINING $Q_1$ , POOLED Q AND STANDARD DEVIATION

#### C.1 EQUATIONS FOR $Q_1$ AND POOLED Q

Equations for determining  $Q_1$  and pooled Q are derived in this section.  $Q_1$  and Q are empirical-based crack growth parameters in a crack size-time relationship.  $Q_1$  describes the crack growth for a single crack in a data set, and Q describes the typical crack growth for a complete fractographic data set. Since Q is used to characterize the typical crack growth for all cracks in a data set, it is referred to as a "pooled Q" value.

A suitable crack size-time relationship is needed to back-extrapolate fractographic results to time zero and/or to grow on EIFSD forward to make durability analysis predictions. A useful crack size-time relationship, given in Eq. C-1, has been derived from the crack growth law given in Eq. C-1 [13]. In eqs. C-1 and C-2,  $a(t)$  = crack size at any time t,  $a(0)$  = crack size at  $t = 0$  or EIFS, Q = crack growth parameter (empirical constant) and  $da(t)/dt$  = crack growth rate. The relationship given in Eqs. C-1 and C-2 have been shown to very reasonable for durability analyses [2,13,15,24-30].

$$a(t) = a(0) \exp(Qt) \quad (C-1)$$

$$da(t)/dt = Qa(t) \quad (C-2)$$

For a single crack application the notations in Eqs. C-1 and C-2 are modified as shown in Eqs. C-3 and C-4.

$$a(t) = a(0) \exp(Q_1 t) \quad (C-3)$$

$$da(t)/dt = Q_1 a(t) \quad (C-4)$$

$Q_1$  in Eq. C-3 can be determined using a least square fit procedure and fractographic data or analytical predictions based on a LEFM crack growth program [e.g., 38].

Equation C-3 can be transformed into a linear least square fit form by taking the natural log of both sides of Eq. C-3 as shown in Eq. C-5. A least squares fit expression

$$\frac{\ln a(t)}{Y_i} = \frac{\ln a(0)}{B} + Q_1 \frac{t}{X_i} \quad (C-5)$$

for  $Q_1$  is given in Eq. C-6, where  $N$  = number of  $X, Y$  pairs

$$Q_1 = \frac{N \sum_{i=1}^N X_i Y_i - \sum_{i=1}^N X_i \sum_{i=1}^N Y_i}{N \sum_{i=1}^N X_i^2 - \left( \sum_{i=1}^N X_i \right)^2} \quad (C-6)$$

considered in the least squares fit.  $X_i$  and  $Y_i$  are defined in Eq. C-5.

The expression for  $Q$  or "pooled  $Q$ " is derived as follows. An expression for sum squared error (SEE),  $E^2$ , based on Eq. C-2, is given in Eq. C-7. In Eq. C-7,

$$SSE = E^2 = \sum_{i=1}^N \left[ \ln \left( da(t)/dt \right)_i - \ln Q - \ln a_i(t) \right]^2 \quad (C-7)$$

$B = \ln Q$ , and  $N$  = number of cracks in the data set. After minimizing  $E^2$  with respect to  $B$  (i.e.,  $\partial E^2 / \partial B = 0$ ) and simplifying Eq. C-8 is obtained. The resulting expression

$$N \ln Q = \sum_{i=1}^N \left[ \ln \left( \frac{da(t)}{dt} \right)_i - \sum_{i=1}^N \ln a_i(t) \right] \quad (C-8)$$

for  $Q$  is given in Eq. C-9. Fractographic data can be used to define the  $da(t)/dt$  and  $a(t)$  values in Eq. C-9.

$$Q = \exp \left[ \frac{\sum_{i=1}^N \ln \left( \frac{da(t)}{dt} \right)_i - \sum_{i=1}^N \ln a_i(t)}{N} \right] \quad (C-9)$$

An expression for  $Q$  can be derived from Eq. C-9 in terms of  $Q$  as follows. Assume: (1) the fractographic data to be used to determine  $Q$  covers the selected crack size range AL-AU and (2) each crack in the data set is homogeneous. The fatigue cracks are made "homogeneous" by artificially equally spacing the  $a(t)$ 's for each crack in the data set. Thus, each fatigue crack in the data set has an equal number of equally-spaced  $a(t)$ 's in the AL-AU range. We will show that it doesn't matter how many equally spaced cracks are used.

The terms of the numerator of Eq. C-9 can be generalized to sum results for individual fatigue cracks in the data set as follows. In Eqs. C-10 through C-12,  $a_i(t)$  = crack size for crack  $i$  at time  $t$  and  $Q_i$  = crack growth parameter for a single crack as defined by Eq. C-6. The desired expression for  $Q$ , given in Eq. C-13, is obtained by substituting Eq. C-12 into Eq. C-9.

$$\sum_{i=1}^N \ln \left( \frac{da(t)}{dt} \right)_i - \sum_{i=1}^N \ln a_i(t) = \sum_{i=1}^M \sum_{j=1}^N \ln Q_i a_i(t_j) - \sum_{i=1}^M \sum_{j=1}^n a_i(t_j) \quad (C-10)$$

$$= \sum_{i=1}^M \sum_{j=1}^n \ln Q_i + \sum_{i=1}^M \sum_{j=1}^n \ln a_i(t_j) - \sum_{i=1}^M \sum_{j=1}^n \ln a_i(t_j) \quad (C-11)$$

$$= \sum_{i=1}^M \ln Q_i \quad (C-12)$$

$$Q = \exp \left[ \frac{1}{M} \sum_{i=1}^M \ln Q_i \right] \quad (C-13)$$

Equation C-13 is very convenient for computing  $Q$  for a given fractographic data set. In other words,  $Q$  is equal to the exponential of the average  $\ln Q_i$  for the data set.

Equation C-13 was derived assuming that each fatigue crack in the data set had an equal number of equally-spaced  $a(t)$ s per crack. This is equivalent to assuming that  $n$  is fixed for the data set. Equation C-13 is also valid when  $n$  varies for each fatigue crack in the data set. For example, in Eqs. C-10 and C-11, substitute  $n_i$  for  $N$  in this case, where  $n_i$  = the number of  $a(t)$ s for the  $i$ th crack. With this substitution, the last two terms in Eq. C-11 still cancel out leaving the term shown in Eq. C-12.

#### C-2 EQUATION FOR STANDARD DEVIATION

When the stochastic crack growth approach (SCGA) is used (see Section III) the standard deviation,  $\sigma_z$ , is required. Expressions for  $\sigma_z$  are derived and discussed in this section.

Equation C-2 is transformed by taking the natural log of each side of the equation and the result is given in Eq. C-14.

$$\ln da(t)/dt = \ln Q + \ln a(t) \quad (C-14)$$

Suppose  $\ln da(t)/dt$  versus  $\ln a(t)$  is plotted based on the fractographic results for a given data set. Equation C-14 can be used to determine the relationship between

$da(t)/dt$  and  $a(t)$  and is a "best fit" straight line as illustrated in Fig. C.1. By definition,  $\sigma_z$ , is equal to the sum squared error (SSE) between the straight line given in Eq. C-14 divided by the number of  $da(t)/dt$ ,  $a(t)$  pairs in the data set. The SSE in this case is given in Eq. C-15. In Eq. C-15,  $N$  = total number of  $da(t)/dt$ ,  $a(t)$  pairs in the data set. An expression for  $\sigma_z$ , obtained using Eq. C-15, is given in Eq. C-16. In Eq. C-16 the summation is from  $i = 1$  to  $N$ ,

$$SSE = \sum_{i=1}^N \left[ \ln da(t)/dt - \ln Q - \ln a(t) \right]^2 \quad (C-15)$$

$$\begin{aligned} \sigma_z &= \sqrt{SSE/N} \\ &= \sqrt{\frac{1}{N} \sum_{i=1}^N \left[ \ln \left( da(t)/dt \right)_i - \ln Q - \ln a_i(t) \right]^2} \end{aligned} \quad (C-16)$$

where  $N$  is the total number of  $a(t)$  points in the data set or in the selected crack size range, AL-AU.

Equation C-16 can be simplified to obtain an expression for  $\sigma_z$  in terms of  $Q_1$  and  $Q$ . To simplify, Eq. C-4 is substituted into Eq. C-16 and a summation notation is used to obtain the expressions for  $\sigma_z$  as shown in Eq. C-17.

$$\sigma_z = \sqrt{\frac{1}{M} \sum_{i=1}^M \left[ \ln \left( Q_i/Q \right) \right]^2} \quad (C-17)$$

In Eq. C-17,  $M$  = number of cracks in the data set.  $Q_1$  for each crack can be obtained using Eq. C-6, which accounts for the number of  $a(t)$ ,  $t$  pairs in the selected AL-AU range for a given crack.

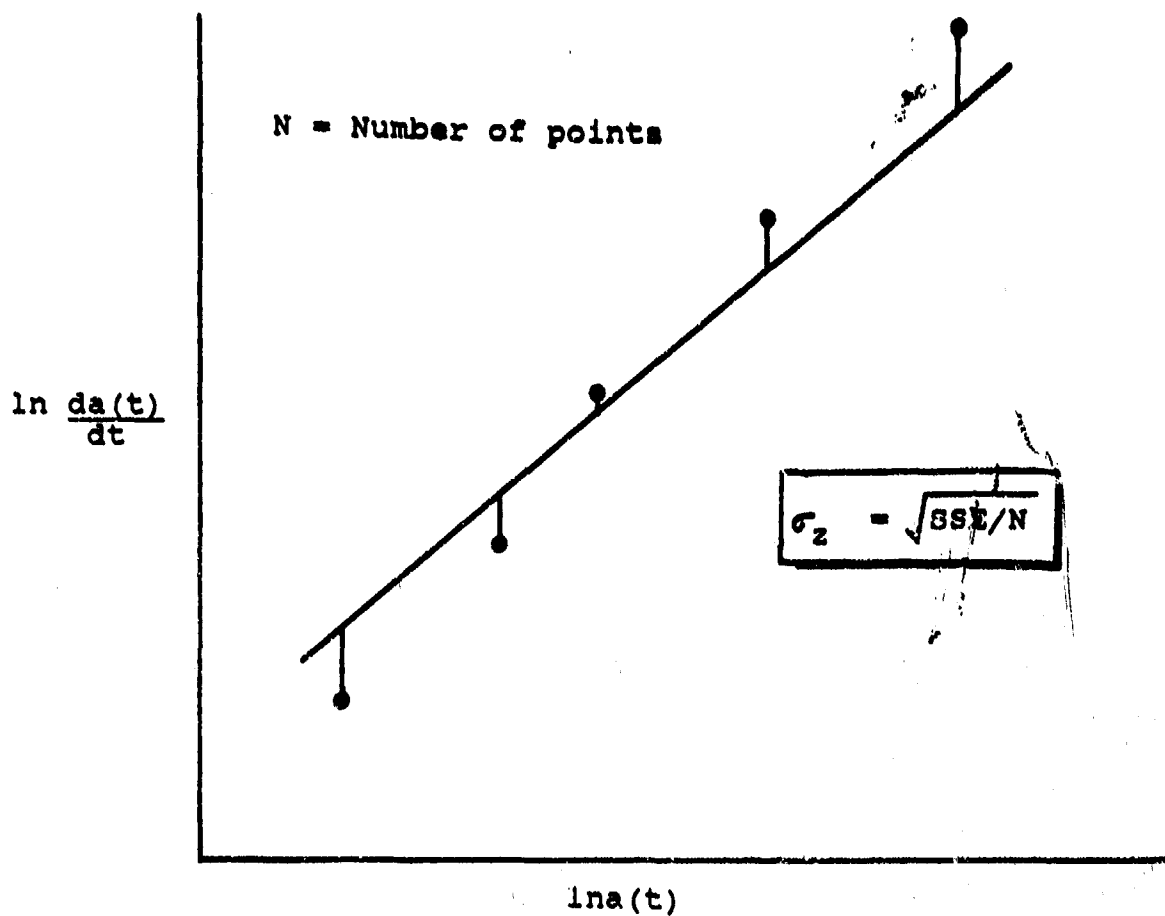


Figure C.1 Conceptual Illustration for Standard Deviation  $\sigma_z$ .

## APPENDIX D

### DURABILITY ANALYSIS EQUATIONS BASED ON SELECTED EIFSD FUNCTIONS AND DCGA

#### D.1 INTRODUCTION

Expression for the cumulative distribution of crack size at anytime  $t$ ,  $F_a(t)(x)$ , and the cumulative distribution of TTCI,  $F_T(t)$  are presented in this section for these different EIFSD functions: (1) two-parameter Weibull, (2) lognormal and (3) lognormal compatible [12].

Expressions for  $F_a(t)(x)$  and  $F_T(t)$  are obtained using a transformation of the applicable EIFSD function,  $F_a(0)(x)$ , and the deterministic crack size-time ultimately given in Eq. D-1. Equation D-1

$$EIFS = x = x_1 \exp(-Qt) \quad (D-1)$$

is based on the crack growth rate model of Eq. C-2 and the generalized crack size-time relationship of Eq. C-1 in which  $a(t) = x_1$  and  $a(0) = EIFS = x$ . In Eq. D-1,  $t$  = time or TTCI corresponding to crack size  $x_1$  and  $Q$  = empirical crack growth constant. The cumulative distribution of EIFS is separately represented by a two-parameter Weibull, a lognormal and a lognormal compatible distribution function. Details are provided in the following.

#### D.2 TWO-PARAMETER WEIBULL EIFSD FUNCTION

The two-parameter Weibull distribution can be used to represent the cumulative distribution of EIFS, denoted by  $F_a(0)(x)$ , as given in Eq. D-2. In Eq. D-2,  $x$  = EIFS random variable,  $\alpha$  and  $\beta$  are empirical constants.



$$F_{\alpha 0}(x) = 1 - \exp \left\{ - \left( \frac{x}{\beta} \right)^{\alpha} \right\} \quad (D-2)$$

The cumulative distribution of crack size at any time  $t$  can be obtained through a transformation of Eq. D-2 and the crack size-time relationship given in Eq. D-1 as follows. In Eq. D-3, the terms are defined as given in Eqs. D-1 and D-2.

$$\begin{aligned} F_{\alpha t}(x) &= F_{\alpha 0} \left[ x = x_1 \exp(-Qt) \right] \\ &= 1 - \exp \left\{ - \left[ \frac{x_1 \exp(-Qt)}{\beta} \right]^{\alpha} \right\} \end{aligned} \quad (D-3)$$

An expression for the cumulative distribution of TTCI can be obtained from Eq. D-3 as follows.

$$\begin{aligned} F_T(t) &= 1 - F_{\alpha t}(x) \\ &= \exp \left\{ - \left[ \frac{x_1 \exp(-Qt)}{\beta} \right]^{\alpha} \right\} \end{aligned} \quad (D-4)$$

The two parameters Weibull function for the cumulative distribution of EIFS, given in Eq. D-2, contains two "EIFSD parameters" to be determined (i.e.,  $\alpha$  and  $\beta$ ). These same parameters are also included in Eqs. D-3 and D-4. Therefore, the EIFSD parameters can be determined using either Eq. D-2, D-3 or D-4. Since the two-parameter Weibull EIFSD function does not have an EIFS upper bound limit ( $x_u$ ), infinite initial flaw sizes are possible.

### D.3 LOGNORMAL EIFSD FUNCTION

The cumulative distribution of EIFS based on the lognormal distribution function is given in Eq. D-5, where:  $\Phi$  = standard normal distribution function,  $x$  = EIFS random variable;  $\mu$  and  $\sigma$  are empirical constants.

$$F_{\alpha 0}(x) = \Phi \left\{ \frac{\ln x - \mu}{\sigma} \right\} \quad (D-5)$$

An expression for the cumulative distribution of crack size at any time  $t$  can be obtained through a transformation of Eq. D-5 using Eq. D-1. The terms in Eq. D-6 are defined in Eqs. D-1 and D-5.

$$F_{a(t)}(x) = \Phi \left\{ \frac{|\ln x_1 - Q_t| - \mu}{\sigma} \right\} \quad (D-6)$$

The cumulative distribution of TTCI can be obtained using Eq. D-7 as follows.

$$\begin{aligned} F_T(t) &= 1 - F_{a(t)}(x) \\ &= 1 - \Phi \left\{ \frac{|\ln x_1 - Q_t| - \mu}{\sigma} \right\} \end{aligned} \quad (D-7)$$

The lognormal EIFSD function in Eq. D-5 contains two EIFSD parameters to be determined (i.e.,  $\mu$  and  $\sigma$ ). Since these same parameters are also included in Eqs. D-6 and D-7, either Eq. D-5, D-6, or D-7 can be used to estimate the EIFSD parameters. Since there is no EIFS upper bound limit,  $x_u$ , for the lognormal EIFSD function, infinite EIFS values are possible.

#### D.4 LOGNORMAL COMPATIBLE EIFSD FUNCTION

The lognormal compatible EIFSD is given in Eq. D-8, in which  $x_u$  = EIFS upper bound limit,  $\mu$  and  $\sigma$  are empirical constants,  $\Phi$  = standard normal distribution function and  $x$  = EIFS random variable. Equation D-8 was obtained by

$$\begin{aligned} F_{a(0)}(x) &= 1 - \Phi \left\{ \frac{[\ln \ln(X_u/X) - \ln Q] - \mu}{\sigma} \right\} \quad 0 < x \leq x_u \\ &= 1.0 \quad x \geq x_u \end{aligned} \quad (D-8)$$

transforming the lognormal cumulative distribution of TTCI using the crack size-time relationship given in Eq. D-1 [12].

An expression for the cumulative distribution of crack size at any time  $t$ , denoted by  $F_a(t)(x)$ , can be obtained through a transformation of Eq. D-8 using Eq. D-1. The resulting expression is given in Eq. D-9, where:  $x_1$  = reference crack size for TTCI,  $t$  = TTCI;  $x_u$ ,  $\mu$  and  $\sigma$  are defined in Eq. D-8.

$$F_{a(t)}(x) = 1 - \Phi \left\{ \frac{\ln \left[ \ln \left( X_u / X_1 \right) + Qt \right] - \ln Q - \mu}{\sigma} \right\} \quad (D-9)$$

An expression for the cumulative distribution of TTCI, denoted by  $F_T(t)$ , can be obtained as follows. All terms in Eq. D-10 have already been defined.

$$\begin{aligned} F_T(t) &= 1 - F_{a(t)}(x) \\ &= \Phi \left\{ \frac{\ln \left[ \ln \left( X_u / X_1 \right) + Qt \right] - \ln Q - \mu}{\sigma} \right\} \end{aligned} \quad (D-10)$$

The lognormal compatible function for the cumulative distribution of EIFS, Eq. D-8, contains three "EIFSD parameters" to be determined (i.e.,  $x_u$ ,  $\mu$  and  $\sigma$ ). The same EIFSD parameters are also included in Eqs. D-9 and D-10 for the cumulative distribution of crack size and of TTCI, respectively. Therefore, the EIFSD parameters can be determined using either Eq. D-8, D-9 or D-10.

## APPENDIX E

### METHODS FOR DETERMINING POOLED Q AND EIFSs FOR DCGA AND SCGA

#### E.1 INTRODUCTION

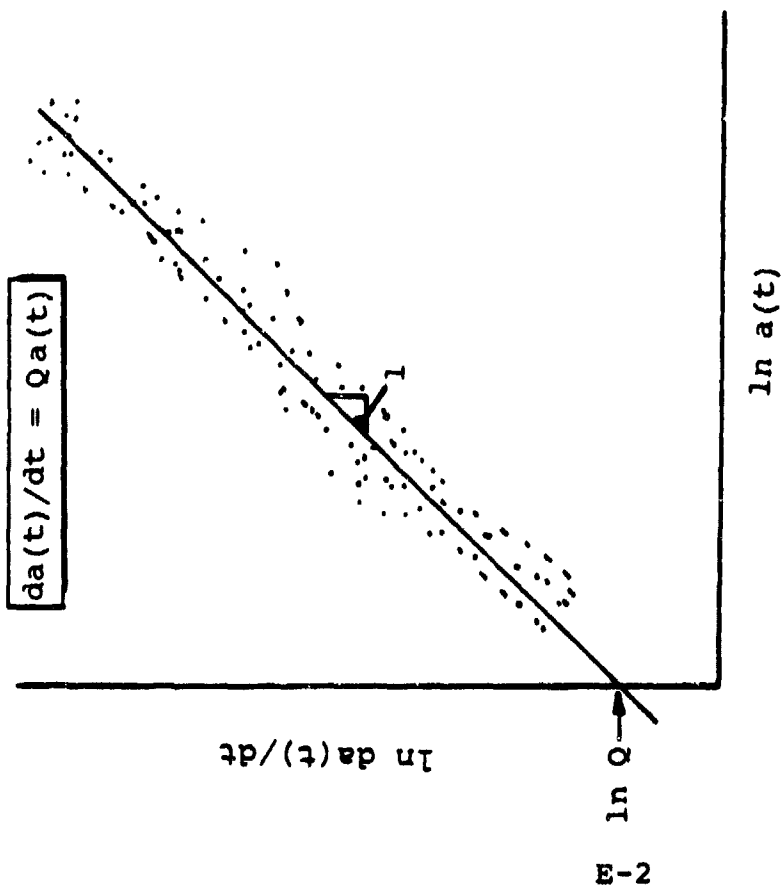
The purpose of this section is to review: (1) different methods and variation for determining the "pooled Q" value for the crack growth rate model shown in Eq. E-1, (2) approaches for determining DCGA-based and (3) approaches for determining SCGA-based EIFSs. In Eq. E-1,  $da(t)/dt$  = crack

$$da(t)/dt = Q a(t) \quad (E-1)$$

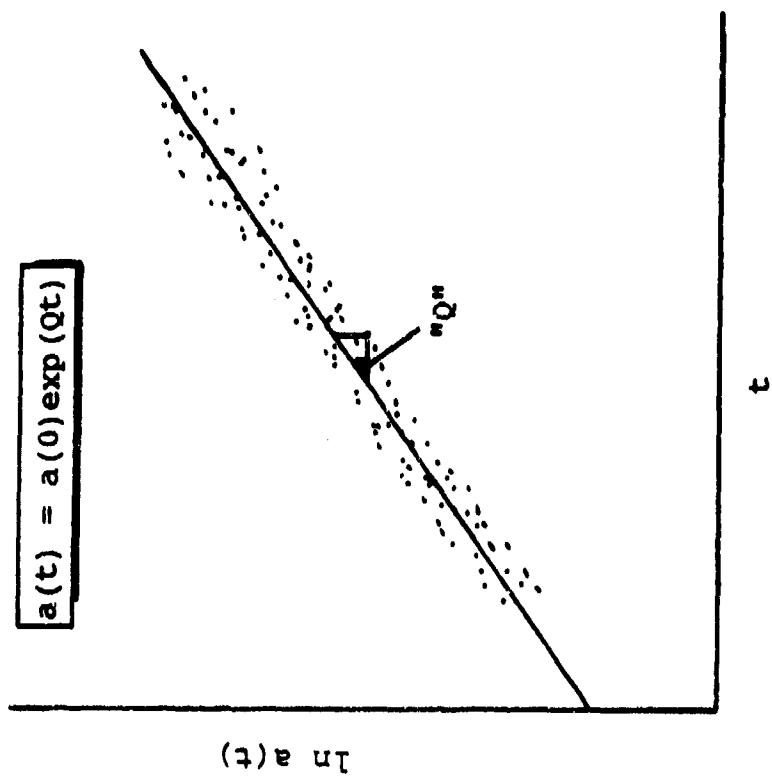
growth rate,  $Q$  = empirical-based crack growth rate constant and  $a(t)$  = crack size at time  $t$ . Methods are reviewed and discussed for durability analysis applications.

#### E.2 APPROACHES FOR DETERMINING POOLED Q

Two different approaches were investigated for estimating the "pooled Q" value for a given fractographic data set: (1) direct use of the crack growth rate model given in Eq. E-1 and (2) use of the generalized crack size-time relationship given in Eq. C-1. The two approaches are conceptually compared in Fig. E.1. The  $Q$  parameter in Eqs. E-1 and C-1 is an empirical constant usually based on fractographic results.  $Q$  can be estimated using a least squares fit procedure using either Eq. E-1 or C-1. Both equations are considered separately in the following.



(a) Least Square Fit Based on Crack Growth Rate Model



(b) Least Square Fit Based on Crack Size-Time Relationship

Figure E.1. Conceptual Comparisons of Two Approaches for Determining the "Pooled  $Q$ " Value for a Given Fractographic Data Set.

### E.2.1 Q Based on $da(t)/dt$

Q can be determined from Eq. E-1 using the transformation shown in Eq. E-2, the least squares fit procedure and fractographic data. The crack growth rate,  $da(t)/dt$ , for any  $a(t)$

$$\ln da(t)/dt = \ln Q + \ln a(t) \quad (E-2)$$

can be estimated from the fractographic data (i.e.,  $a(t)$  versus  $t$  results) using either the modified secant method [10] or the five-point incremental polynomial method [12]. Q in Eq. E-2 can be determined using Eq. E-3 which is based on the least squares fit procedure. The following subscripted

$$Q = \exp \left\{ \frac{\sum_{i=1}^M \sum_{j=1}^{N_i} \ln \left[ da(t)/dt \right]_{ij} - \sum_{i=1}^M \sum_{j=1}^{N_i} \ln a_i(t_j)}{\sum_{i=1}^M N_i} \right\} \quad (E-3)$$

subscripted notation is used in Eq. E-3:  $M$  = number of fatigue cracks in the data set;  $N_i$  = number of  $a(t)$ 's for the  $i$ th fatigue crack; subscripts  $i$  and  $j$  denote the  $i$ th crack and  $j$ th value;  $a_i(t_j)$  =  $j$ th value of crack size for the  $i$ th fatigue crack; and  $[da(t)/dt]_{ij}$  =  $j$ th crack growth rate for the  $i$ th fatigue crack. Equation E-3 can be interpreted as follows. The "pooled Q" value for a given data set depends on the crack growth rates for each  $a(t)$  for each fatigue crack in the fractographic data set.

To be consistent, Q should be determined using the fractographic data in a selected fractographic crack size range, AL-AU. The  $da(t)/dt$  data in the AL-AU range can be used to compute Q various ways: (1) use only the actual  $da(t)/dt$ ,  $a(t)$  data that lies in the AL-AU range for each fatigue crack

in the data set, and (2) artificially equalize the number of  $da(t)/dt$ ,  $a(t)$ 's in the AL-AU range. With the first approach there may not be an equal number of  $a(t)$ 's for each fatigue crack in the data set in the AL-AU range. This means that each fatigue crack may not be equally weighted in the determination of  $Q$ . One way to equally weight each fatigue crack is to equalize the number of  $a(t)$ 's in the selected AL-AU range. All fatigue cracks in the data set may not cover the same crack size range. Also, the smallest and largest  $a(t)$  in the AL-AU range will probably be different for each fatigue crack. The influence of the crack size range and the fatigue crack weighting on the resulting  $Q$  has been investigated (see Volume II [ 3]).

#### E.2.2 $Q$ Based on Crack Size-Time Relationship

$Q$  can also be determined for a given fractographic data set using the crack size-time relationship given in Eq. C-1. The main advantage of this approach is that " $Q$ " can be determined directly from the fractographic results (i.e.,  $a(t)$  versus  $t$ ) without computing  $da(t)/dt$ 's. Equation C-1 can be transformed into a linear least squares fit form as shown in Eq. 40. Then,  $Q$  can be determined using Eq. C-14, based on the least squares fit procedure. In Eq. E-4, the subscripts  $i$  and  $j$

$$Q = \frac{\left( \sum_{i=1}^M N_i \right) \left( \sum_{i=1}^M \sum_{j=1}^{N_i} X_{ij} Y_{ij} \right) - \left( \sum_{i=1}^M \sum_{j=1}^{N_i} X_{ij} \right) \left( \sum_{i=1}^M \sum_{j=1}^{N_i} Y_{ij} \right)}{\left( \sum_{i=1}^M N_i \right) \left( \sum_{i=1}^M \sum_{j=1}^{N_i} X_{ij}^2 \right) - \left( \sum_{i=1}^M \sum_{j=1}^{N_i} X_{ij} \right)^2} \quad (E-4)$$

denote the  $i$ th fatigue crack and the  $j$ th  $a(t), t$  values for the  $i$ th fatigue crack  $M$  = total number of fatigue cracks in the fractographic data set,  $N_i$  = total number of  $a(t)$ 's for the  $i$ th fatigue crack;  $X_{ij}$  and  $Y_{ij}$  are defined in Eq. E-5. In Eq. E-5,  $t_{ij}$  =  $j$ th time for the  $i$ th fatigue crack and  $a_i(t_j)$  =  $j$ th crack size for the  $i$ th fatigue crack.

$$\begin{aligned} X_{ij} &= t_{ij} \\ Y_{ij} &= \ln a_i(t_j) \end{aligned} \quad (E-5)$$

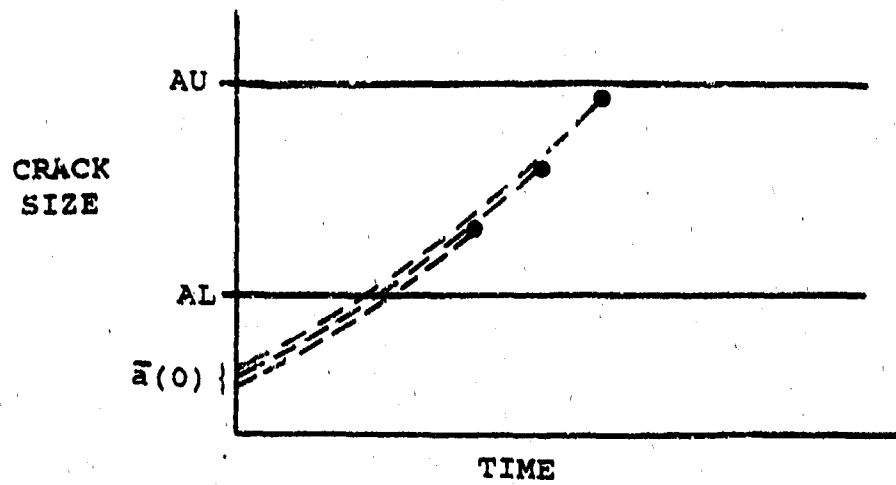
Using Eq. E-4, the "pooled  $Q$ " value for a given fractographic data set can be determined using the  $a(t), t$  values for each fatigue crack in the data set. This approach is recommended for durability analyses. Whatever approach is used to compute  $Q$ , it is important to recognize that " $Q$ " depends on the fractographic crack size range used.

### E.3 APPROACHES FOR DETERMINING DCGA-BASED EIFSS

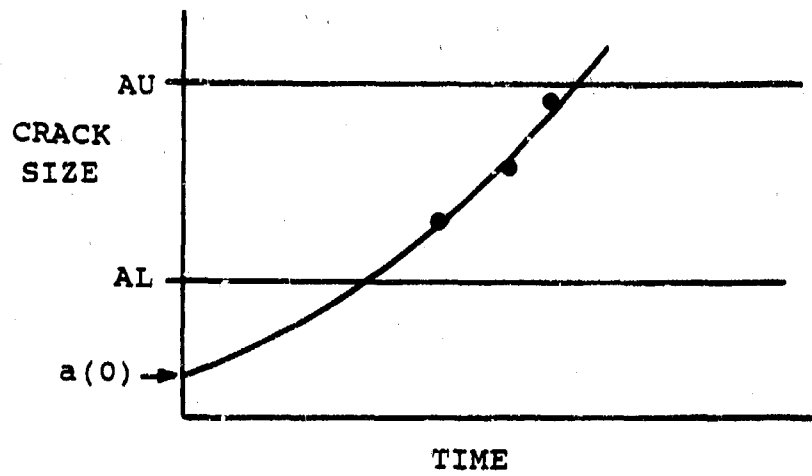
The following three different approaches for determining EIFSSs based on the DCGA were investigated: (1) averaging method, (2) least squares fit method, and (3) use "pooled  $Q$ " and selected  $x_1, t$ . A conceptual comparison of the three approaches is shown in Fig. E.2.

The "pooled  $Q$ " value for a given fractographic data set is required to implement each of the three approaches. Methods for computing "pooled  $Q$ " are described in Section E.2. EIFSSs for each of the three approaches should be determined using the same fractographic crack size range, AL-AU, used to determine the "pooled  $Q$ " value. Each of the three approaches for computing DCGA-based EIFSSs are described and discussed in the following.

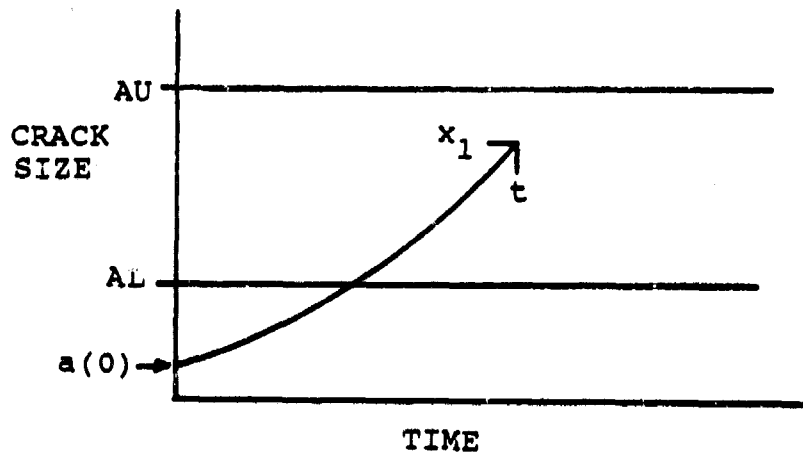




(a) Averaging Method



(b) Least Squares Fit Method



(c) Use Pooled  $Q$  and Selected  $x_1, t$

Figure E.2. Deterministic Crack Growth Approaches for Determining EIFSs.

### E.3.1 Averaging Method

The "averaging method" is based on the  $a(t)$  relationship given in Eq. E-6.

$$a(t) = X_1 \exp(-Qt) \quad (E-6)$$

This equation is obtained from Eq. C-1 by substituting  $a(t) = x_1$  and solving for  $a(t)$ . The crack size-time relationship given in Eq. E-6 is used to compute the average EIFS,  $\bar{a}(0)$ , for each fatigue crack in the data set as follows. An EIFS is computed for each  $a(t), t$  pair in the selected AL-AU range for a given fatigue crack. Then, an average EIFS is computed. This approach is mathematically described in Eq. E-7.

$$\text{Average EIFS} = \bar{a}(0) = \frac{1}{N} \sum_{i=1}^N a(t_i) \exp(-Qt_i) \quad (E-7)$$

In Eq. E-7,  $N$  = total number of  $a(t), t$  pairs in the selected AL-AU crack size range,  $a(t_i)$  =  $i$ th crack size,  $t_i$  =  $i$ th time, and  $Q$  = "pooled  $Q$ " value for given data set (see Section E.2).

### E.3.2 Least Squares Fit Method

The EIFS for a given fatigue crack is determined as follows. The crack size-time relationship given in Eq. E-6 is transformed into a linear least squares fit form as shown in Eq. C-14. An expression for  $a(0)$  or EIFS, given in Eq. E-8, is obtained using Eq. C-14 and a least squares fit procedure. In Eq. E-8,  $N$  = total number of  $a(t), t$  pairs in the

$$a(0) = \text{EIFS} = \exp \left\{ \frac{\sum_{i=1}^N Y_i - Q \sum_{i=1}^N X_i}{N} \right\} \quad (E-8)$$

selected AL-AU crack size range and  $Q$  = "pooled  $Q$  value" for a given fractographic data set.  $X_1$  and  $Y_1$  are defined in Eq. E-9, where:

$$X_1 = t_1 \quad (E-9)$$

$$Y_1 = \ln a(t_1)$$

$t_1$  =  $i$ th time corresponding to the crack size  $a(t_1)$  in the selected AL-AU crack size range.

### E.3.3 Selected $x_1, t$

This approach is a simple variation of the averaging method described in Section E.3.1. For example, the EIFS for a given fatigue crack can be determined directly from Eq. E-6 for selected  $x_1$  and  $t$  values (e.g., see Fig. E.2(c)). This approach is convenient to use when the EIFSD parameters are to be estimated using a "TTCI fit." In this case, the TTCIs for a given data set are defined for a selected reference crack size,  $x_1$ . Then, the EIFSs for each fatigue crack in the data set can be determined using Eq. E-6.

### E.3.4 Discussion

The three methods for determining EIFSs based on the DCGA are evaluated in Volume II [3]. No significant differences in comparable EIFS values were found for any of the three methods described in this section. Whatever method is used to determine the EIFSs, the resulting EIFSs values will depend on the crack growth rate model used as well as the fractographic crack size range used. Therefore, a different set of EIFSs will be obtained for each crack growth rate model and AL-AU crack size range used.

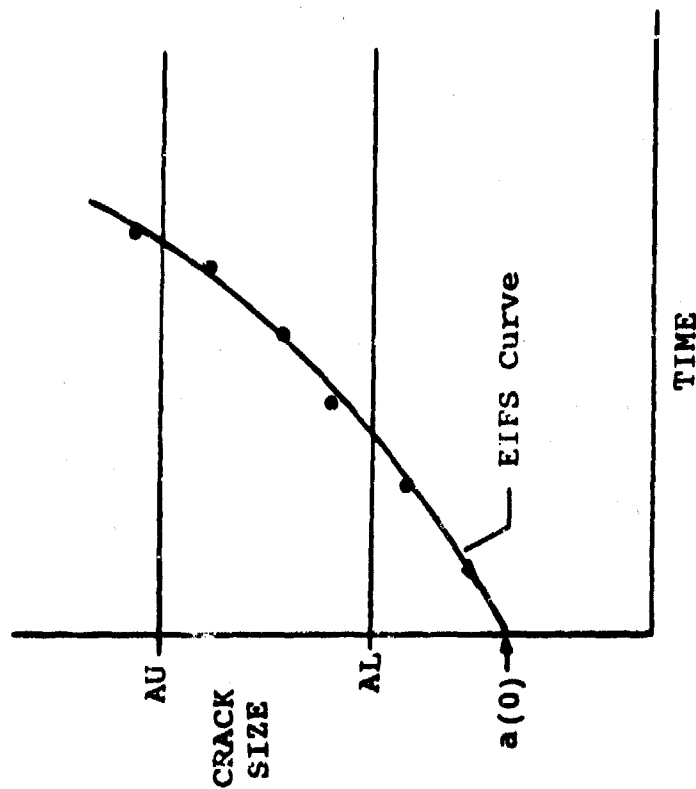
Different crack growth rate models could be used to determine the EIFSs. The simple model described in Eq. E-1 has been found to be very reasonable for describing the crack size-time relationship for different materials, load spectra, stress levels, fastener hole type (i.e., straight bore or countersunk), % bolt load transfer and stress level [2,3,10,15, 24-30].

#### E.4 APPROACHES FOR DETERMINING SCGA-BASED EIFSs

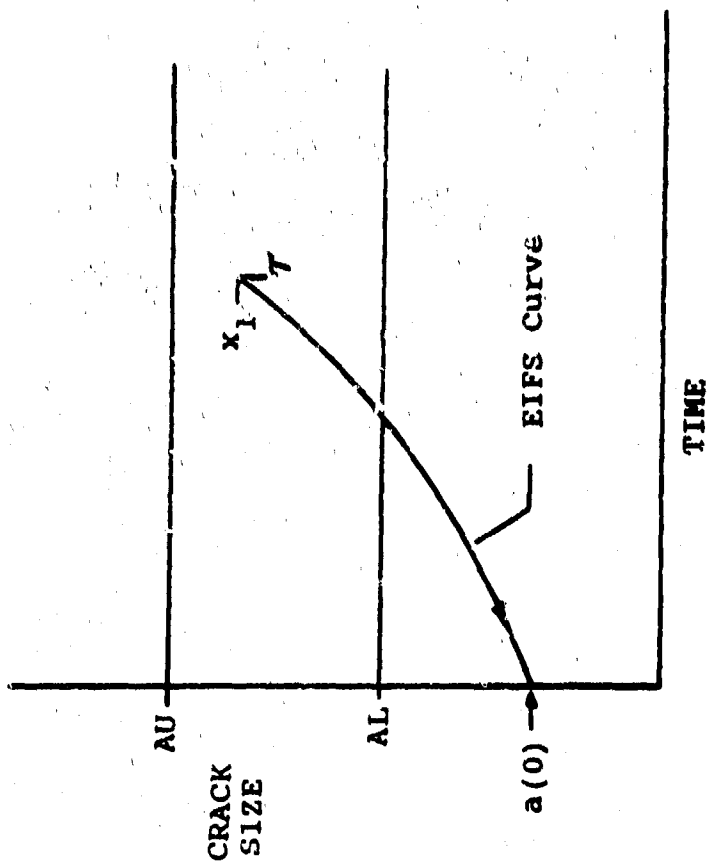
Procedures for determining SCGA-based EIFSs are described and discussed in this section. Deterministic and stochastic-based EIFSs can be determined from the same fractographic data set as illustrated in Fig. E.3. For the DCGA, the EIFSs are determined using the same EIFS master curve. On the other hand, SCGA-based EIFSs are based on a separate EIFS curve for each fatigue crack in the fractographic data set. DCGA and SCGA based EIFSs are discussed in Section A.3 and the two approaches are conceptually compared in Fig. A.2.

SCGA-based EIFSs are used in the durability analysis extension described in Section III. With the SCGA, each fatigue crack in the fractographic data set is back-extrapolated to time zero using the fractographic results for each crack (separately) and a reasonable crack growth rate model, such as that described in Section III.

The crack size-time relationship shown in Eq. C-3 can be used to compute the SCGA-based EIFS as follows. Equation C-3 can be interpreted as follows for the SCGA:  $a(t)$  = crack size at any time  $t$ ,  $a(0)$  = SCGA-based EIFS, and  $Q_i$  = empirical crack growth constant for the  $i$ th fatigue crack. Equation C-3 is transformed into a linear least squares fit form as shown in Eq. C-5 so that  $a(0)$  and  $Q_i$  can be determined. The empirical constant  $Q_i$  can be estimated using Eq. C-6. Then,  $a(0)$  can be determined using either Eq. C-3 or Eq. E-6 (with  $Q = Q_i$ ) as conceptually described in Fig. A.2. Like



(a) Least Square Fit for  $a(0)$



(b) Given  $Q$ , Determine  $a(0)$  for Given  $x_1, \tau$

Figure E.3. Conceptual Comparison of Two Different Approaches for Determining SCGA-Based EIFS.

the DCGA, the SCGA-based EIFSs are determined using a selected fractographic crack size range (e.g., AL-AU = 0.01" - 0.05"). To be consistent, the  $Q_1$  value for each fatigue crack must reflect the same AL-AU range.

The "averaging method" described in Section E.3.1 can also be used to compute SCGA-based EIFSs. For example, in Eq. E-7 simply use the crack growth rate parameter for a given crack,  $Q_1$ , instead of the "pooled Q" value for the complete fractographic data set.

It is shown in Volume II [ 3 ] that it doesn't make a lot of difference whether Eq. C-3, E-6 or E-7 is used to determine the EIFSs. The EIFS values for a given fatigue crack may vary depending which equation is used to determine the EIFS. However, the net effect on the EIFSD parameters and respective initial flaw sizes for a given probability is not significant.

Once the SCGA-based EIFSs have been determined, the EIFSD parameters can be estimated using EIFS data and the CLSSA described in Appendix F.

## APPENDIX F

### APPROACHES FOR ESTIMATING EIFSD PARAMETERS

#### F.1 INTRODUCTION

Five approaches are described and discussed in this section for estimating the EIFSD parameters for selected  $F_a(0)(x)$  functions including: (1) combined least square sums approach (CLSSA), (2) method of moments (MM), (3) homogeneous EIFS approach (HEIFS), (4) upper tail fit and (5) non-linear approach. The five different approaches are discussed in terms of the Weibull compatible EIFSD function. The CLSSA is also described for the lognormal compatible EIFSD function.

#### F.2 COMBINED LEAST SQUARE SUMS APPROACH (CLSSA)

The CLSSA is described and discussed in this section for two different EIFSD functions: (1) Weibull compatible and (2) lognormal compatible. This approach can be used to estimate the EIFSD parameters, in a global sense, for one or more fractographic data sets. Since the effects of scaling are accounted for, fractographic data sets can be "mixed and matched" to estimate the EIFSD parameters. A step-by-step procedure for implementing the approach is presented.

The EIFSD parameters for a given EIFSD function can be estimated using either TTCI or EIFS input data for one or more fractographic data sets. The TTCIs or EIFSs (whichever is used) for each fractographic data set are ranked and least square sums are obtained for each data set using the applicable EIFSD function and data used to fit the EIFSD parameters. Each least square sum accounts for the scaling factor

for each data set. The combined least square sums for all data sets are obtained by summing the applicable value for each data set. Then, the EIFSD parameters are estimated using the combined least square sums and a global fit.

#### F.2.1 General Procedure

The procedure for implementing the CLSSA is given below:

1. Select fractographic data sets to be used in the pooling procedure to estimate the EIFSD parameters in a global sense.
2. Acquire the TTCIs or EIFSs for each data set to be used to estimate the EIFSD parameters.
3. Define scaling factor,  $\ell_i$ , for each data set to be used.
4. Select the EIFSD function to be used and then transform it into a linear least square fit form. If EIFSs are used for the fit, use the expression for the cumulative distribution of EIFS for the fit. On the other hand, if TTCIs are used for the fit use the expression for the cumulative distribution of TTCI based on the transformation of the EIFSD and the crack growth model used (e.g., Eq. C-2).
5. Assume a value for the EIFS upper bound within the constraint: largest EIFS in any data set  $\leq x_u \leq 0.05"$ . Compute the combined least square sums for all data sets combined. Use these results to estimate the EIFSD parameters.



6. The EIFSD parameters can be optimized within the  $x_u$  constraint. A reasonable way to do this is to minimize the total standard error for all data sets used to determine the EIFSD parameters. The optimum  $x_u$  can be determined by trial and error until the total standard error has been minimized.

### F.2.2 Weibull Compatible Function

The purpose of this section is to describe how the EIFSD parameters can be estimated for the Weibull compatible EIFSD function using the CLSSA. Equations are presented and discussed for estimating the EIFSD parameters based on "TTCI fit" and on "EIFS fit." The effects of scaling are accounted for.

F.2.2.1 EIFS Fit. The Weibull compatible EIFSD function  $F_{a(0)}(x)$ , is given in Eq. F-1, where  $x_u$  = upper bound EIFS limit,

$$F_{a(0)}(x) = \exp \left\{ - \left[ \frac{\ln(x_u/x)}{\phi} \right]^\alpha \right\}; 0 \leq x \leq x_u \quad (F-1)$$

$$= 1.0 \quad ; x \geq x_u$$

$x$  = EIFS variable, and  $\alpha$  and  $\phi$  are empirical constants. Equation F-1 is modified to account for scaling as shown in Eq. F-2. In Eq. F-2,  $F_{a_l(0)}(x)$  = cumulative distribution of maximum EIFS based on the largest fatigue crack per specimen in 1 of  $l$  fastener holes. Other terms in Eq. F-2 have already been defined. Since  $F_{a_l(0)}(x) = F_{a(0)}(x)$  when there is no scaling, Eq. F-2 is a general expression that applies to cases where  $l \geq 1$ .

$$F_{a_l(0)}(x) = \exp \left\{ -l \left[ \frac{\ln(x_u/x)}{\phi} \right]^\alpha \right\}; l \geq 1 \quad (F-2)$$

The EIFSD parameters in Eq. F-2 (i.e.,  $x_u$ ,  $\alpha$  and  $\phi$ ) can be estimated using the CLSSA as follows. Transform Eq. F-2 into the linear least square fit form given in Eq. F-3. For data pooling purposes the subscripted notation shown in Eq. F-4 is used.

$$\ln \left[ - (1/l) \ln F_{a_i(0)}(x) \right] = \alpha \ln \ln (x/x_u) - \alpha \ln \phi \quad (F-3)$$

$$\left. \begin{aligned} X_{ij} &= \ln \ln (x/x_{ij}) \\ Y_{ij} &= \ln \left[ - (1/l_i) \ln F_{a_i(0)}(x_{ij}) \right] \\ B &= -\alpha \ln \phi \\ F_{a_i(0)}(x_j) &= j/(N_i + 1); j = 1, 2, \dots, N_i \end{aligned} \right\} \quad (F-4)$$

In Eq. F-4,  $l_i$  = number of fastener holes for the replicate specimen in the  $i$ th data set,  $x_{ij}$  =  $j$ th EIFS values for the  $i$ th data set,  $x_u$  = EIFS upper bound limit,  $N_i$  = number of EIFS values for the  $i$ th data set,  $F_{a_i(0)}(x_{ij})$  = cumulative distribution of maximum EIFS based on the largest fatigue crack per specimen for the  $i$ th data set.

The EIFSD parameters  $\alpha$  and  $\phi$  in Eq. F-3 can be estimated for a given  $x_u$  value using a least squares fit procedure. Expressions for estimating  $\alpha$  and  $\phi$  using the CLSSA are shown in Eq. F-5 and F-6, respectively.

$$\alpha = \frac{\sum_{i=1}^M N_i \sum_{j=1}^{N_i} X_{ij} Y_{ij} - \sum_{i=1}^M \sum_{j=1}^{N_i} X_{ij} \sum_{i=1}^M \sum_{j=1}^{N_i} Y_{ij}}{\sum_{i=1}^M N_i \sum_{j=1}^{N_i} X_{ij}^2 - \left[ \sum_{i=1}^M \sum_{j=1}^{N_i} X_{ij} \right]^2} \quad (F-5)$$

$$\phi = \exp \left\{ \frac{\alpha \sum_{i=1}^M \sum_{j=1}^{N_i} X_{ij} - \sum_{i=1}^M \sum_{j=1}^{N_i} Y_{ij}}{\alpha \sum_{i=1}^M N_i} \right\} \quad (F-6)$$

In Eqs. F-5 and F-6,  $M$  = number of EIFS data sets used to estimate the EIFSD parameters,  $N_i$  = number of EIFSs in the  $i$ th data set. Equations F-5 and F-6 can also be used when a TTCI fit is used. If an EIFS fit is used, then  $X_{ij}$  and  $Y_{ij}$  are defined as shown in Eq. F-4.

#### EIFS Fit Procedure

1. Select EIFS data sets to be used to estimate the EIFSD parameters  $\alpha$  and  $\phi$ .
2. Rank EIFSs in ascending order for each data set separately. Use  $F_{a_{\ell_i}(0)}(x) = j/(N_i + 1)$ .
3. Assume value of  $x_u$  within constraint: largest EIFS in any data set  $\leq x_u \leq 0.05$ ". Assume the maximum  $x_u$  corresponds to the initial flaw size used for damage tolerance requirements.
4. Compute the least square sums shown in Eqs. F-5 and F-6 for each EIFS data set separately, including the appropriate scaling factor,  $\ell_i$ , for each data set.
5. Combine the least square sums for each EIFS data set into a total sum for all data sets to be used in the pooling procedure.
6. Use the iterative procedure described in Fig. 9 to optimize the EIFSD parameters. The total standard error, TSE, for all EIFS data sets can be determined using Eq. F-7. The subscripted notation and terms in Eq. F-7 are defined the same way as given in Eqs. F-4 through F-6.

$$TSE = \sqrt{\frac{\sum_{i=1}^M \sum_{j=1}^{N_i} \left\{ \frac{j}{(N_i + 1)} - \exp \left\{ -\ell_i \left[ \frac{\ln(x_u/x_j)}{\phi} \right] \alpha \right\} \right\}^2}{\sum_{i=1}^M N_i}} \quad (F-7)$$

F.2.2.2 TTCI Fit. The cumulative distribution of TTCI,  $F_T(t)$ , is given in Eq. F-8.

$$F_T(t) = 1 - F_{a(t)}(x) = 1 - \exp \left\{ - \left[ \frac{\ln(x_u/x_1) + Qt}{\phi} \right]^\alpha \right\} \quad (F-8)$$

In Eq. F-8,  $x_u$  = EIFS upper bound limit,  $x_1$  = reference crack size for TTCIs,  $t$  = TTCI,  $Q$  = empirical-based crack growth rate parameter, and  $\alpha$  and  $\phi$  are empirical constants.

The EIFSD parameters for the Weibull compatible EIFSD function can be determined using a "TTCI fit" as follows. First, the expression for the cumulative distribution of TTCI, given in Eq. F-8 is modified to account for scaling as shown in Eq. F-9. In Eq. F-9,  $F_{T_l}(t)$  = cumulative distribution of minimum TTCI based on the largest fatigue crack in  $l$  of  $l$  holes per specimen. Other terms in Eq. F-9

$$\begin{aligned} F_{T_l}(t) &= 1 - F_{a_l(t)}(x) \\ &= 1 - \exp \left\{ - l \left[ \frac{\ln(x_u/x_1) + Qt}{\phi} \right]^\alpha \right\}; l \geq 1 \end{aligned} \quad (F-9)$$

have already been defined. Since  $F_{T_l}(t) = F_T(t)$  when  $l = 1$  (no scaling), Eq. F-9 applies for  $l \geq 1$ .

Equation F-9 can be transformed into a linear least square fit form as shown in Eq. F-10.

$$\ln \left\{ (1/D) \ln \left[ 1 - F_{T_l}(t) \right] \right\} = \alpha \ln \left[ \ln(x_u/x_1) + Qt \right] - \alpha \ln \phi \quad (F-10)$$

To implement the data pooling procedure, the terms in Eq. F-10 are expressed in a subscripted form as shown in Eq. F-11. In Eq. F-11,  $\ell_i$  = number of fastener holes for the replicate specimen in the  $i$ th data set,  $N_i$  = number of TTCI values for the  $i$ th data set,  $x_u$  = EIFS upper bound limit,  $x_1 = a_0$  = reference crack size for TTCIs,  $Q_i$  = pooled  $Q$  value for the  $i$ th data set,  $t_{ij}$  =  $j$ th TTCI value for the  $i$ th data set,  $F_{T_{\ell_i}}(t_{ij})$  =  $j$ th value for the cumulative distribution of TTCI minimums (i.e., based on the largest fatigue crack per specimen in each data set) for the  $i$ th data set.

$$\left. \begin{aligned} X_{ij} &= \ln \left[ \ln (x_u/x_1) + Q_i t_{ij} \right] \\ Y_{ij} &= \ln \left\{ - (1/\ell_i) \ln \left[ 1 - F_{T_{\ell_i}}(t_{ij}) \right] \right\} \\ B &= -\alpha \ln \phi \\ F_{T_{\ell_i}}(t_{ij}) &= j/(N_i + 1); j = 1, 2, \dots, N_i \end{aligned} \right\} \quad (F-11)$$

The EIFSD parameters  $\alpha$  and  $\phi$  in Eq. F-9 can be determined for a selected  $x_u$  value using Eqs. F-5, F-6, and F-11.

#### TTCI Fit Procedure

1. Select TTCI data sets to be used to estimate the EIFSD parameters  $\alpha$  and  $\phi$ .
2. Rank the TTCIs in ascending order for each data set separately. Use  $F_{T_{\ell_i}}(t_{ij}) = j/(N_i + 1)$ .
3. Assume value of  $x_u$  with constraint: largest EIFS in any data set  $\leq x_u \leq 0.05$ ".
4. Compute the least square sums in Eq. F-5 and F-6 for each TTCI data set separately, including the applicable scaling factor,  $\ell_i$ , for each data set. Use the terms defined in Eq. F-11.
5. Combine the least square sums for each TTCI data set into a total sum for all data sets to be used in the pooling procedure.

6. Use the iteration procedure described in Fig. 9 to optimize the EIFSD parameters. The total standard error, TSE, for all TTCI data sets can be determined using Eq. F-12.

$$TSE = \sqrt{\frac{\sum_{i=1}^M \sum_{j=1}^{N_i} \left\{ j/(N_i + 1) - 1 + \exp \left\{ -l_i \left[ \frac{\ln(x_u/x_i) + Q_i t_{ij}}{\phi} \right] \right\}^2 \right\}}{\sum_{i=1}^M N_i}} \quad (F-12)$$

In Eq. F-12, M = number of fractographic data sets used in the pooling procedure.

### F.2.3 Lognormal Compatible EIFSD Function

The purpose of this section is to describe how the EIFSD parameters can be estimated for the lognormal compatible function using the CLSSA. Equations are presented and discussed for estimating the EIFSD parameters based on an "EIFS fit" or "TTCI fit." The effects of scaling are accounted for.

F.2.3.1 EIFS Fit. The cumulative distribution of EIFS for the lognormal compatible distribution is given in Eq. F-13, where

$$F_{a(0)}(x) = 1 - \phi \left\{ \frac{\ln \ln(x_u/x) - \ln Q}{\sigma} - \mu \right\} \quad (F-13)$$

$\phi$  = standard normal distribution function,  $x_u$  = EIFS upper bound limit,  $x$  = EIFS variable,  $Q$  = empirical crack growth rate constant, and  $\mu$  and  $\sigma$  are empirical constants.

Equation F-13 is modified to account for scaling as shown in Eq. F-14. Equation F-14 is transformed into a linear least squares fit form as shown in Eq. F-15.

$$F_{a_i(0)}(x) = \left\{ 1 - \phi \left\{ \frac{\ln \ln(x_u/x) - \ln Q}{\sigma} - \mu \right\} \right\}^{1/N_i} \quad (F-14)$$

$$\phi^{-1} \left\{ 1 - \left[ F_{a_i(0)}(x) \right]^{N_i} \right\} = \left[ \ln \ln(x_u/x) - \ln Q \right] / \sigma - \mu / \sigma \quad (F-15)$$

For data pooling purposes, the terms in Eq. F-15 are represented in a subscripted form as shown in Eq. F-16. In Eq. F-16, the subscripts  $i$  and  $j$  refer to the  $i$ th EIFS data set and the  $j$ th EIFS value for the  $i$ th data set,  $Q_i$  = "pooled  $Q$ " value for the  $i$ th data set,  $\phi^{-1}$  = inverse of the standard normal distribution function, and  $N_i$  = number of EIFSs in the  $i$ th data set.

$$\begin{aligned} X_{ij} &= \ln \ln(x_u/x_{ij}) - \ln Q_i \\ Y_{ij} &= \phi^{-1} \left\{ 1 - \left[ F_{a_i(0)}(x_{ij}) \right]^{N_i} \right\} \\ F_{a_i(0)}(x_{ij}) &= j/(N_i + 1); j = 1, 2, \dots, N_i \end{aligned} \quad (F-16)$$

The EIFSD parameters  $\mu$  and  $\sigma$  in Eq. F-14 and F-15 can be estimated for a given  $x_u$  using the CLSSA. Expressions for  $\mu$  and  $\sigma$  are given in Eqs. F-17 and F-18, respectively. The subscripted notations in Eqs. F-17 and F-18 are the same as those in Eqs. F-5 and F-6 for a global fit.

$$\sigma = \frac{\sum_{i=1}^M N_i \sum_{j=1}^{N_i} X_{ij}^2 - \left[ \sum_{i=1}^M \sum_{j=1}^{N_i} X_{ij} \right]^2}{\sum_{i=1}^M N_i \sum_{j=1}^{N_i} X_{ij} Y_{ij} - \sum_{i=1}^M \sum_{j=1}^{N_i} X_{ij} \sum_{i=1}^M \sum_{j=1}^{N_i} Y_{ij}} \quad (F-17)$$

$$\mu = \frac{\sum_{i=1}^M \sum_{j=1}^{N_i} X_{ij} - \sigma \sum_{i=1}^M \sum_{j=1}^{N_i} Y_{ij}}{\sum_{i=1}^M N_i} \quad (F-18)$$

## EIFS Fit Procedure

The step-by-step procedure for estimating the EIFSD parameters using an "EIFS fit" is the same as that given in section F.2.2.1 except as follows:

1. In step 4 compute the least square sums for Eqs. F-17 and F-18 using Eq. F-16.
2. In step 6 determine the total standard error, TSE, using Eq. F-19.

$$TSE = \sqrt{\frac{\sum_{i=1}^M \sum_{j=1}^{N_i} \left\{ \frac{j}{(N_i+1)} - \left[ 1 - \phi \left\{ \frac{\ln \ln(x_u/x_l) - \ln Q_i}{\sigma} \right\} \right] \right\}^2}{\sum_{i=1}^M N_i}} \quad (F-19)$$

**F.2.3.2 TCI Fit.** The cumulative distribution of TCI,  $F_T(t)$ , is shown in Eq. F-20. To account for "scaling" Eq. F-20 is modified as shown in Eq. F-22. For data pooling purposes, a subscripted notation is used as shown in Eq. F-22.

$$\begin{aligned} F_T(t) &= 1 - F_{a_l(t)}(x) \\ &= \phi \left\{ \frac{\ln \left[ \ln(x_u/x_l) + Qt \right] - \ln Q - \mu}{\sigma} \right\} \end{aligned} \quad (F-20)$$

$$\begin{aligned} F_T(t) &= 1 - F_{a_l(t)}(x) \\ &= 1 - \left\{ 1 - \phi \left\{ \frac{\ln \left[ \ln(x_u/x_l) + Qt \right] - \ln Q - \mu}{\sigma} \right\} \right\}^l \end{aligned} \quad (F-21)$$

$$\phi^{-1} \left\{ 1 - \left[ 1 - F_{T_l}(t) \right]^{1/l} \right\} = \frac{1}{\sigma} \left\{ \ln \left[ \ln(x_u/x_l) + Qt \right] - \ln Q \right\} - \mu/\sigma \quad (F-22)$$



$$\left. \begin{aligned} X_{ij} &= \ln \left[ \ln(x_u/x_i) + Q_i t_{ij} \right] - \ln Q_i \\ Y_{ij} &= \Phi^{-1} \left\{ 1 - \left[ 1 - F_{T_{ij}}(Q_i) \right]^{1/\mu_i} \right\} \\ F_{T_{ij}}(Q_i) &= j/(N_i + 1); j = 1, 2, \dots, N_i \end{aligned} \right\} \quad (F-23)$$

In Eq. F-23, the subscripts  $i$  and  $j$  refer to the  $i$ th TTCI data set and the  $j$  the TTCI value for the  $i$ th data set,  $Q_i$  = "pooled  $Q$ " value for the  $i$ th fractographic data set,  $t_{ij}$  =  $j$ th TTCI value for the  $i$ th data set,  $\mu_i$  = scaling factor for the  $i$ th data set,  $\Phi^{-1}$  = inverse of the standard normal distribution function and  $N_i$  = number of TTCIs in the  $i$ th data set.

The EIFSD parameters  $\mu$  and  $\sigma$  in Eq. F-21 and F-22 can be estimated for a given  $x_u$  using the CLSSA. Using the expressions of Eq. F-23, and can be determined using Eqs. F-17 and F-18, respectively.

#### TTCI Fit Procedure

The step-by-step procedure for estimating the EIFSD parameters using a "TTCI fit" is the same as that given in section F.2.2.2 except as follows:

1. In step 4 compute the least square sums for Eqs. F-17 and F-18 using Eq. F-19.

2. In step 6 determine the total standard error. (TSE) using Eq. F-24.

$$TSE = \sqrt{\frac{\sum_{i=1}^M \sum_{j=1}^{N_i} \left\{ \frac{1}{(N_i + 1)} - 1 + \left\{ 1 - \Phi \left[ \frac{\ln \left[ \ln(x_u/x_i) + Q_i t_{ij} \right] - \ln Q_i - \mu}{\sigma} \right] \right\}^{\mu_i} \right\}^2}{\sum_{i=1}^M N_i}} \quad (F-24)$$

### F.3 METHOD OF MOMENTS APPROACH

The method of moments can be used to estimate the constants  $\alpha$  and  $\beta$  in the two-parameter Weibull cumulative distribution function shown in Eq. F-25 [e.g., 61]. In Eq. F-25,  $x$  is a random variable;  $\alpha$  and  $\beta$  are empirical constants. The method of moments approach can also be used to estimate the EIFSD parameters for the Weibull compatible EIFSD function using either a "EIFS fit" or a "TTCI fit." The method of moments approach accounts for scaling of different fractographic data sets and can be used to estimate the EIFSD parameters for one or more data sets in a global sense.

$$F(x) = 1 - \exp \left\{ - (x/\beta)^\alpha \right\} \quad (F-25)$$

The usual way the constants  $\alpha$  and  $\beta$  in Eq. F-25 are determined by the method of moments is as follows:

1. Compute the mean of the random variable,  $x$ , using Eq. F-26, where  $N$  = number of  $x_i$  values in the sample.

$$\text{Mean}(\bar{x}) = \frac{\sum_{i=1}^N x_i}{N} \quad (F-26)$$

2. Compute the standard deviation of  $x$  using Eq. F-27.

$$S(\bar{x}) = \sqrt{\frac{\sum_{i=1}^N x_i^2}{N} - \left( \frac{\sum_{i=1}^N x_i}{N} \right)^2} \quad (F-27)$$

3. Compute the coefficient of variation of  $x$  using Eq. F-28.

$$\text{COV}(\bar{x}) = \frac{\text{Mean}(\bar{x})}{S(\bar{x})} \quad (F-28)$$

4.  $\alpha$  can be determined from the expression for  $\text{COV}(x)$ , given in Eq. F-29, where  $\Gamma()$  = standard gamma function.

$$COV(\bar{x}) = \sqrt{\frac{\Gamma(1 + 2/\alpha)}{\Gamma^2(1 + 1/\alpha)} - 1} \quad (F-29)$$

A table of  $COV(\bar{x})$  versus  $\alpha$  values can be obtained from Eq. F-29 using assumed  $\alpha$  values.  $\alpha$  can then be determined for the given  $COV(\bar{x})$  by interpolating the results in the table of  $COV(\bar{x})$  versus  $\alpha$  values.

5. The constant  $\beta$  in Eq. F-25 is determined from Eq. F-30.

$$\beta = \frac{Mean(\bar{x})}{\Gamma(1 + 1/\alpha)} \quad (F-30)$$

The above general procedure can be used to estimate the EIFSD parameters for the Weibull compatible EIFSD function as follows. Expressions for the probability of crack exceedance,  $p(i, T)$ , and the cumulative distribution of TTCI,  $F_{T_\ell}(t)$ , are given in Eqs. F-31 and F-32, respectively. In Eqs. F-31 and F-32,  $\ell$  = scaling factor,  $x_u$  = EIFS upper bound limit,  $\alpha$  and  $\phi$  are empirical constants. In Eq. F-31,  $x$  = EIFS variable and in Eq. F-32,  $x_1$  = reference crack size for TTCI,  $t$ . Equations F-31 and F-32 both account for scaling. In these equations note that:  $F_{a_\ell}(0) = F_a(0) = F_a(0)(x)$  and  $F_{T_\ell}(t) = F_T(t)$  when  $\ell = 1.0$  (no scaling).

$$p(i, T) = 1 - F_{a_\ell(0)}(x) = 1 - \exp \left\{ - \ell \left[ \frac{\ln(x_u/x)}{\phi} \right]^\alpha \right\} \quad (F-31)$$

$$F_{T_\ell}(t) = 1 - F_{a_\ell(t)}(x) = 1 - \exp \left\{ - \ell \left[ \frac{\ln(x_u/x_1) + Q t}{\phi} \right]^\alpha \right\} \quad (F-32)$$

Equation F-31 can be rewritten in an equivalent form as Eq. F-25 as shown in Eq. F-33. Equation F-33 can be used when EIFSs are used to estimate the EIFSD parameters  $\alpha$  and  $\phi$  for a given  $x_u$ . In Eq. F-33, since the term  $\ell^{1/\alpha} \ln(x_u/x)$  is dimensionless the mean of this term can be determined for a given  $x_u$  using all the EIFS data sets combined. An iterative procedure and method of moments approach are described later for estimating  $\alpha$  and  $\phi$  for a given  $x_u$ .

$$p(i, t) = 1 - \exp \left\{ - \left[ \frac{\ell^{1/\alpha} \ln(x_u/x)}{\phi} \right]^\alpha \right\} \quad (F-33)$$

Equation F-32 for the cumulative distribution of TTCI can also be reduced to the same form as Eq. F-25 by imposing the condition that  $x_1 = x_u$ . This condition requires that the reference crack size for TTCIs be equal to the EIFS upper bound limit ( $x_u$ ) used. The resulting expression for  $F_{T_t}(t)$  is given in Eq. F-34. In Eq. F-34, since the term  $\ell^{1/\alpha} Q t$  is dimensionless, the TTCIs for one or more data sets can be used to estimate  $\alpha$  and  $\phi$  for a given  $x_u$  using the method of moments.

$$F_{T_t}(t) = 1 - \exp \left\{ - \left[ \frac{\ell^{1/\alpha} Q t}{\phi} \right]^\alpha \right\} \quad (F-34)$$

The method of moments approach for estimating the EIFSD parameters for the Weibull compatible EIFSD function is schematically illustrated in Fig. F.1. Various aspects and details of this approach will now be discussed for both EIFS and TTCI fits. Either an EIFS or a TTCI fit can be used to estimate the EIFSD parameters for the Weibull compatible EIFSD function.

Let the terms in the numerator of Eqs. F-33 and F-34 be defined as shown in Eqs. F-35 and F-36, respectively.

$$\lambda_{ij} = (\ell_i^{1/\alpha_k}) \ln(x_u/x_j) \quad (F-35)$$

$$\psi_{ij} = (\ell_i^{1/\alpha_k}) Q_i t_j \quad (F-36)$$

A subscripted notation is used for fractographic data pooling and tracking purposes. The subscripts  $i$  and  $j$  denote the  $i$ th fractographic data set and  $j$ th value from the  $i$ th data set. In Eqs. F-35 and F-36,  $\ell_i$  = scaling factor for  $i$ th data set,  $\alpha_k$  = trial value of  $\alpha$  in iterative process,  $x_j$  =  $j$ th value of EIFS in the  $i$ th data set,  $Q_i$  = empirical crack growth constant ("pooled  $Q$ " value) for the  $i$ th data set and  $t_j$  =  $j$ th TTCI value for a given reference crack size  $x_1$  for the  $i$ th data set.

Expressions for the mean, standard deviation and coefficient of variation (COV) for  $\bar{\delta}$  are given in Eqs. F-37, F-38 and F-39, respectively. These equations are written in a subscripted form for data pooling purposes and computer applications. The same general expressions for the mean, standard deviation and COV for  $\bar{\delta}$  can be used for either an EIFS or TTCI fit. For example, if a EIFS fit is used let  $\delta_{ij} = \lambda_{ij}$  and if a TTCI fit is used let  $\delta_{ij} = \psi_{ij}$ . In Eqs.

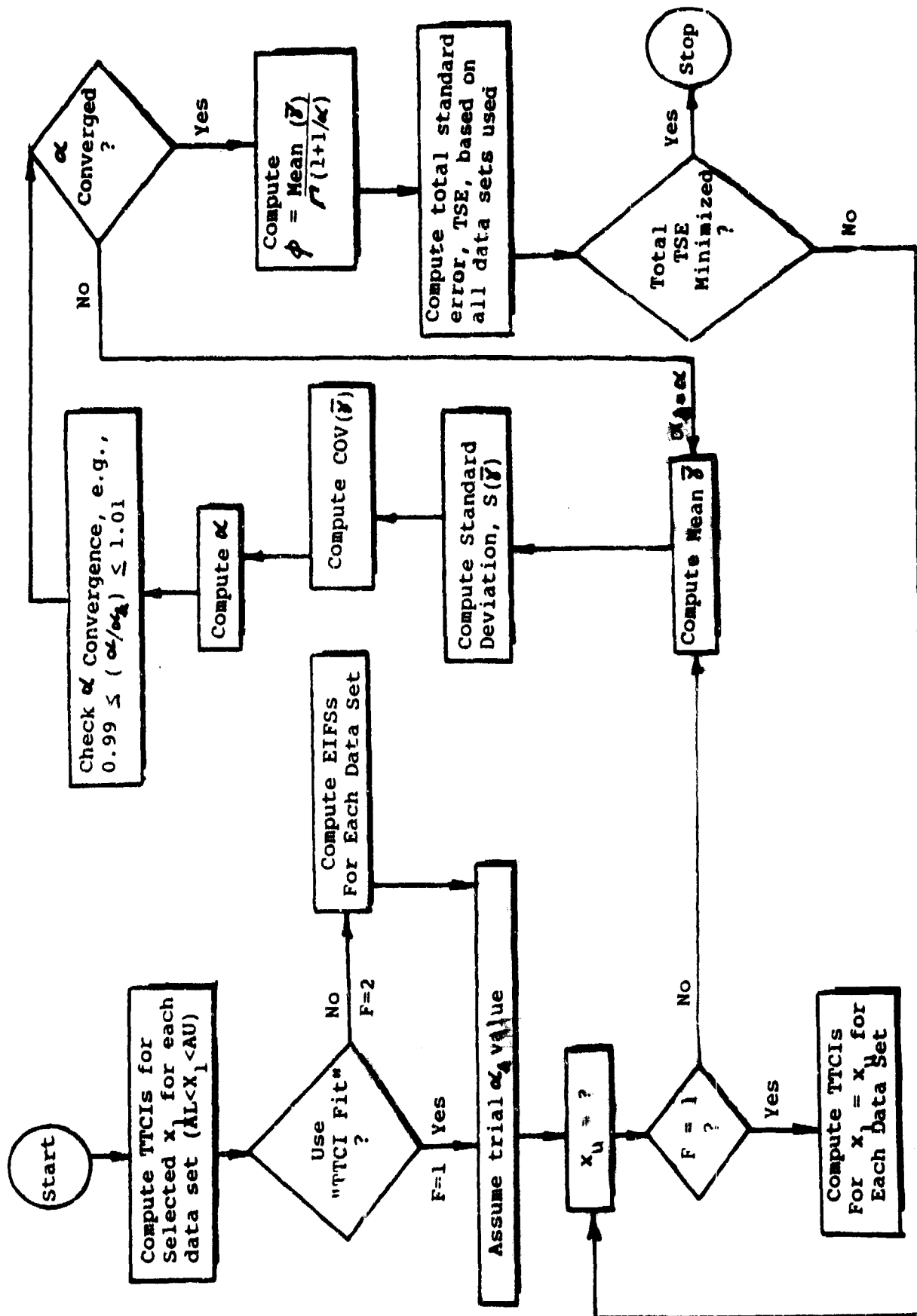


Figure F.1. Schematic for Implementing Method of Moments Approach for Estimating the EIFSD Parameters for the Weibull Compatible EIFSD Function.

F-37 - F-39, the subscripts  $i$  and  $j$  denote the  $i$ th data set and  $j$ th value from the  $i$ th data set,  $M$  = number of data sets used in the pooling procedure and  $N_i$  = number of samples for the  $i$ th data set.

$$Mean(\bar{\gamma}) = \frac{\sum_{i=1}^M \sum_{j=1}^{N_i} \gamma_{ij}}{\sum_{i=1}^M N_i} \quad (F-37)$$

$$S(\bar{\gamma}) = \sqrt{\frac{\sum_{i=1}^M \sum_{j=1}^{N_i} \gamma_{ij}^2}{\sum_{i=1}^M N_i} - \left[ \frac{\sum_{i=1}^M \sum_{j=1}^{N_i} \gamma_{ij}}{\sum_{i=1}^M N_i} \right]^2} \quad (F-38)$$

$$COV(\bar{\gamma}) = \frac{Mean(\bar{\gamma})}{S(\bar{\gamma})} \quad (F-39)$$

Since one of the constants to be determined (i.e.,  $\alpha$ ) is included in Eqs. F-35 and F-36, an iterative technique is used. The general procedure is schematically described in Fig. F.1. Computer software has been developed to implement the method of moments approach described in this section [6].

#### F.4 HOMOGENEOUS EIFS APPROACH

With the homogeneous EIFS (HEIFS) approach EIFSs are computed for each fractographic data set and the resulting EIFSs for all data sets are pooled and ranked into one homo-

geneous EIFS population. The EIFSD parameters can then be estimated using the applicable EIFS population and a least squares fit procedure.

The HEIFS approach presented in this section has two major limitations: (1) it doesn't account for scaling (i.e.,  $k=1$ ) of different fractographic data sets used; and (2) since the EIFSs for all data sets are combined into one homogeneous population, the inherent scatter for each data set is obscured in the process. Scaling of different fractographic data sets can be accounted for using the HEIFS approach and an iterative procedure. However, details are not presented herein. If the HEIFS approach is used to estimate the EIFSD parameters for multiple fractographic data sets, scaling should be accounted for.

The HEIFS approach will be discussed assuming the EIFSD is represented by the Weibull compatible function. The general procedure is conceptually described in Fig. F.2 and in the following.

1. Compute the EIFSs using the DCGA for each fractographic data set to be used to estimate the EIFSD parameters in a global sense.

2. Pool and rank the EIFSs for all data sets into one homogeneous population. Let  $N_i$  = number of EIFSs in the  $i$ th EIFS data set and  $M$  = total number of EIFS data sets to be used. Then, the total number of EIFSs of all data sets combined is denoted by  $n$  as shown in Eq. F-40.

$$n = \sum_{i=1}^M N_i \quad (F-40)$$



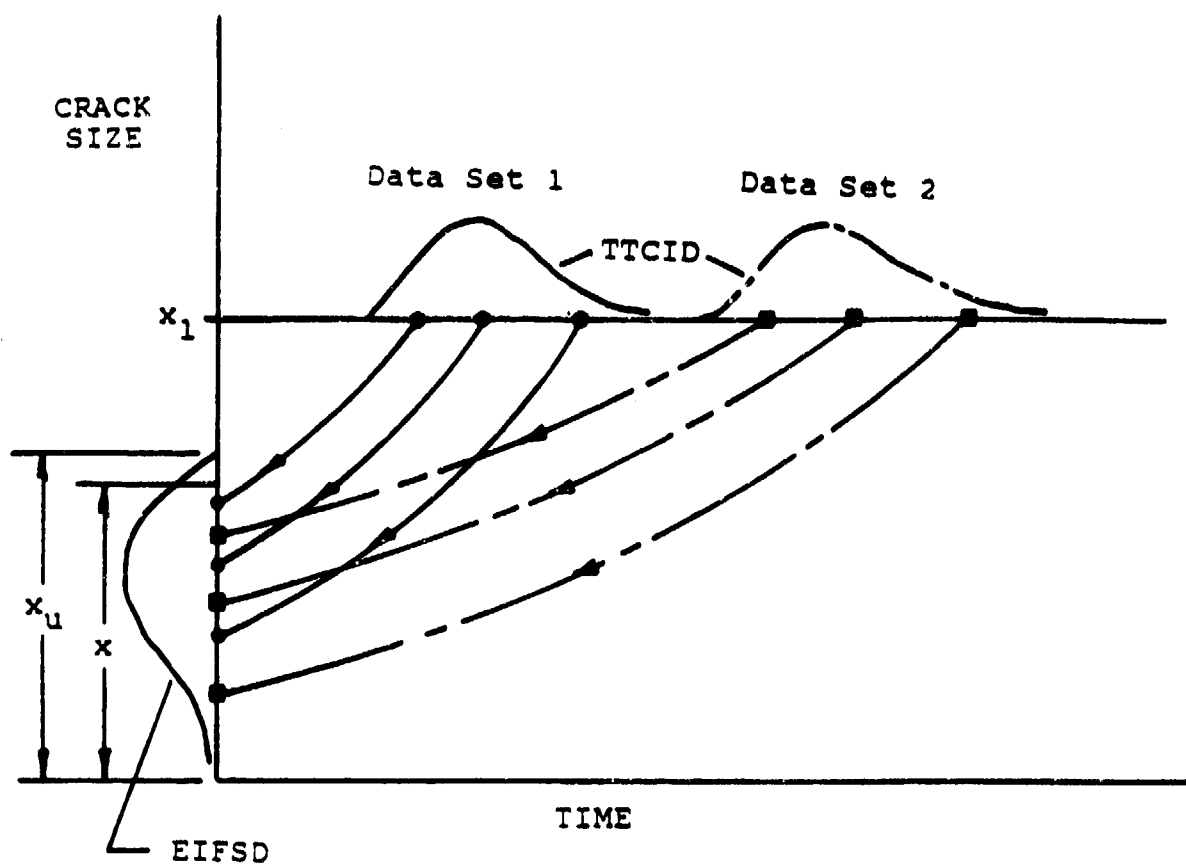


Figure F.2. Conceptual Illustration of the Homogeneous EIFS Approach for Defining Initial Fatigue Quality.

The cumulative distribution of EIFS,  $F_{a(0)}(x)$ , can be determined using Eq. F-41.

$$F_{a(0)}(x) = i/(n+1); i = 1, 2, \dots, n \quad (F-41)$$

3. The Weibull compatible EIFSD function,  $F_{a(0)}(x)$ , given in Eq. F-42, can be transformed into a linear least square fit form as shown in Eq. F-43. In Eq. A-7,  $x$  = EIFS random variable;  $x_u$  = EIFS upper bound limit;  $\alpha$  and  $\phi$  are empirical constants.

$$F_{a(0)}(x) = \exp \left\{ - \left[ \frac{\ln(x_u/x)}{\phi} \right]^\alpha \right\}; 0 \leq x \leq x_u \quad (F-42)$$

$$= 1.0 \quad ; x \geq x_u$$

$$\ln \left\{ -\ln F_{a(0)}(x) \right\} = \alpha \ln \ln(x_u/x) - \alpha \ln \phi \quad (F-43)$$

4. The terms in Eq. F-43 are modified into a subscripted notation to facilitate data pooling and implementation using a computer. Expressions are shown in Eq. F-44.

$$X_i = \ln \ln(x_u/x)$$

$$Y_i = \ln \left\{ -\ln F_{a(0)}(x) \right\} \quad (F-44)$$

$$= \ln \left\{ -\ln \left[ i/(n+1) \right] \right\}; i = 1, 2, \dots, n$$

$$B = -\alpha \ln \phi$$

5. The empirical constants  $\alpha$  and  $\phi$  can be estimated for a given  $x_u$  using the least squares fit procedure as follows.

$$\alpha = \frac{n \sum_{i=1}^n X_i Y_i - \left( \sum_{i=1}^n X_i \right) \left( \sum_{i=1}^n Y_i \right)}{n \sum_{i=1}^n X_i^2 - \left( \sum_{i=1}^n X_i \right)^2} \quad (F-45)$$

$$\phi = \exp \left\{ \frac{\alpha \sum_{i=1}^n X_i - \sum_{i=1}^n Y_i}{\alpha n} \right\} \quad (F-46)$$

6. The EIFSD parameters  $\alpha$  and  $\phi$  can be optimized for a given  $x_u$  by minimizing the total standard error, TSE, within the constraints of  $x_u$  (i.e., largest EIFS in the population  $\leq x_u \leq 0.05$ ). An iterative procedure can be used to minimize the TSE.

#### F.5 UPPER TAIL FIT

For functional impairment due to fuel leakage and ligament breakage, the main interest is in the larger EIFSs in the upper tail of the EIFSD. A method was investigated for estimating the EIFSD parameters using only the largest EIFSs. The interest here was to determine the EIFSD based on an "upper tail fit" and then to determine the initial flaw size corresponding to a selected probability of crack exceedance. The actual study is documented in Volume II [3] and the technical details are given in the following.

Basic assumptions of the upper tail fit approach are:  
 (1) the EIFSD parameters can be estimated using the largest EIFSs in the EIFS population; and (2) the initial flaw size

for a given probability can be obtained from the derived EIFSD and the resulting initial flaw size should be more realistic than if the entire EIFS population is used to estimate the EIFSD parameters.

### Procedure

1. Compute EIFSs for each EIFS data set to be considered in the global fit.

2. Once the EIFSs have been determined, the CLSSA or the HEIFS approach (if  $\ell = 1$  applies) can be used to estimate the EIFSD parameters. If the CLSSA is used, the EIFSs for each data set are ranked in ascending order. A probability of crack exceedance is selected (e.g.,  $p(i, \tau) = 0.30$ ) and ranked EIFSs  $\leq 0.30$  are used to estimate the EIFSD parameters using the CLSSA. If the HEIFS approach is used, all the EIFSs are pooled and ranked into one homogeneous population. A probability of crack exceedance,  $p(i, \tau)$  is selected and the ranked EIFSs in the upper tail of the EIFSD are used to estimate the EIFSD parameters. In this case a least squares fit procedure is used.

3. The EIFSD parameters can be optimized for a given EIFS upper bound limit,  $x_u$ , by minimizing the total standard error for the fit. This can be accomplished using a trail and error procedure as follows: (1) assume  $x_u$  within constraint (i.e., largest EIFS in population  $\leq x_u \leq 0.05"$ ); (2) select  $p(i, \tau)$ ; (3) compute  $\alpha, \phi$  and total standard error, TSE; and (4) repeat (1) to (3) until the TSE is a minimum.

### F.6 NON-LINEAR APPROACH

A non-linear approach for estimating the EIFSD parameters for the Weibull compatible EIFSD function was investigated. To estimate the EIFSD parameters for the Weibull

compatible function using the least squares fit procedure, the appropriate equation must first be transformed into a linear least squares fit form. A  $\ln \ln$  transformation is required. Since the least squares error is measured in terms of the transformed equation, rather than the original equation, there is a tendency to give more weight to the extremal input data than the intermediate data when the linear least squares fit method is used. For this reason, a non-linear approach was investigated. Results are presented in Volume II [3]. The approach is described in this section and the equations are derived for implementing the method.

A general non-linear approach for estimating model constants has been developed [62]. This approach can be used to estimate the EIFSD parameters for the Weibull compatible EIFSD function in a global sense (non-linear fit). Details are as follows.

The cumulative distribution of TTCI based on the Weibull compatible EIFSD function and the deterministic crack growth rate model is given in Eq. F-9.

The "EIFSD parameters" are  $x_u$ ,  $\alpha$  and  $\phi$ . In Eq. F-9, note that:  $F_{T_\ell}(t) = F_T(t)$  when  $\ell = 1$  (no scaling).

The residual equation for Eq. F-9 is given in Eq. F-47,

$$r = F_{T_\ell}(t) \Big|_{\alpha_0, \phi_0} - \Delta \alpha_0 \left( \frac{\partial f}{\partial \alpha} \right)_0 - \Delta \phi_0 \left( \frac{\partial f}{\partial \phi} \right)_0 \quad (F-47)$$

where:  $r$  = residual

$$F_{T_\ell}(t) = 1/(N + 1),$$

$$F_{T_\ell}(t) \Big|_{\alpha_0, \phi_0} = \text{predicted cumulative distribution of TICI based on a selected } x_u \text{ and initial values of } \alpha_0 \text{ and } \phi_0.$$

$$f = F_{T_\ell}(t),$$

$\alpha$  and  $\phi$  are the EIFSD parameters;  $\alpha_0$  and  $\phi_0$  are initial values for  $\alpha$  and  $\phi$ , respectively;  $\Delta\alpha_0$  and  $\Delta\phi_0$  are increments of  $\alpha$  and  $\phi$  to be determined. The partial derivative,  $(\partial f/\partial\alpha)_0$  and  $(\partial f/\partial\phi)_0$ , are given in Eqs. F-48 and F-49, respectively, where  $\gamma$  is defined in Eq. F-50.

$$(\partial f/\partial\alpha)_0 = \ell \exp(-\ell \gamma^{\alpha_0}) (\gamma^{\alpha_0}) (\ell n \gamma) \quad (F-48)$$

$$(\partial f/\partial\phi)_0 = -\alpha_0 \ell \exp(-\ell \gamma^{\alpha_0}) \left\{ \left[ \ell n(X_u/X_1) + Q t \right] \gamma^{\alpha_0} \right\} \left\{ \phi^{-\alpha_0-1} \right\} \quad (F-49)$$

$$\gamma = \ell n \left[ \frac{\ell n(X_u/X_1) + Q t}{\phi} \right] \quad (F-50)$$

In Eq. F-47,  $\Delta\alpha_0$  and  $\Delta\phi_0$  can be determined using the linear least squares fit procedure. The sum squared error,  $E^2$ , for Eq. F-47 is given in Eq. F-51.

$$E^2 = \sum_{i=1}^N \left\{ u(N+1) - F_{T_i}(t) \Big|_{\alpha_0, \phi_0} - \Delta\alpha_0 (\partial f/\partial\alpha)_0 - \Delta\phi_0 (\partial f/\partial\phi)_0 \right\}^2 \quad (F-51)$$

Expressions for  $\Delta\alpha_0$  and  $\Delta\phi_0$  can be obtained by minimizing  $E^2$  (i.e.,  $\partial E^2 / \partial \alpha = 0$  and  $\partial E^2 / \partial \phi = 0$  and then solving two linear simultaneous equations. The resulting expressions for  $\Delta\phi_0$  and  $\Delta\alpha_0$  are given in Eqs. F-52 and F-53, respectively. A subscripted notation is used to facilitate data pooling.

$$\Delta\phi_0 = \frac{\left( \sum_{i=1}^M \sum_{j=1}^{N_i} X_{ij} Y_{ij} \right) \left( \sum_{i=1}^M \sum_{j=1}^{N_i} X_{ij} Z_{ij} \right) - \left( \sum_{i=1}^M \sum_{j=1}^{N_i} X_{ij}^2 \right) \left( \sum_{i=1}^M \sum_{j=1}^{N_i} Y_{ij} Z_{ij} \right)}{\left( \sum_{i=1}^M \sum_{j=1}^{N_i} X_{ij} Y_{ij} \right)^2 - \left( \sum_{i=1}^M \sum_{j=1}^{N_i} X_{ij}^2 \right) \left( \sum_{i=1}^M \sum_{j=1}^{N_i} Y_{ij}^2 \right)} \quad (\text{F-52})$$

$$\Delta\alpha_0 = \frac{\left( \sum_{i=1}^M \sum_{j=1}^{N_i} X_{ij} Z_{ij} \right)}{\sum_{i=1}^M \sum_{j=1}^{N_i} X_{ij}^2} - \Delta\phi_0 \left[ \frac{\sum_{i=1}^M \sum_{j=1}^{N_i} X_{ij} Y_{ij}}{\sum_{i=1}^M \sum_{j=1}^{N_i} X_{ij}^2} \right] \quad (\text{F-53})$$

The  $X_{ij}$ ,  $Y_{ij}$ ,  $Z_{ij}$ , and  $\gamma_{ij}$  terms in Eqs. F-52 and F-53 are defined in Eqs. F-54, F-55, F-56, and F-57, respectively.

$$X_{ij} = l_i \exp(-l_i \gamma_{ij}^{\alpha_0}) \gamma_{ij}^{\alpha_0} \ln \gamma_{ij} \quad (\text{F-54})$$

$$Y_{ij} = -\alpha_0 l_i \exp(-l_i \gamma_{ij}^{\alpha_0}) [\ln(X_u/X_1) + Q_i t_j] \gamma_{ij}^{\alpha_0} (\phi_0)^{-\alpha_0-1} \quad (\text{F-55})$$

$$Z_{ij} = j/(N_i+1) - 1 + \exp(-l_i \gamma_{ij}^{\alpha_0}) \quad (\text{F-56})$$

$$\gamma_{ij} = \ln \left[ \frac{\ln(X_u/X_1) + Q_i t_j}{\phi} \right] \quad (\text{F-57})$$

In Eqs. F-54 through F-57, the subscripts  $i$  and  $j$  denote the  $i$ th TTCI data set and the  $j$ th value from the  $i$ th data set,  $\alpha_i$  = scaling factor for the  $i$ th TTCI data set,  $Q_i$  = pooled  $Q$  value for the  $i$ th fractographic data set,  $t_j$  =  $j$ th TTCI value in the  $i$ th data set; other terms have already been defined.

A schematic of the non-linear procedure for estimating/optimizing the EIFSD parameters for one or more TTCI data sets is shown in Fig. F.3. This general procedure can be extended to an "EIFS fit" but appropriate equations would have to be developed.



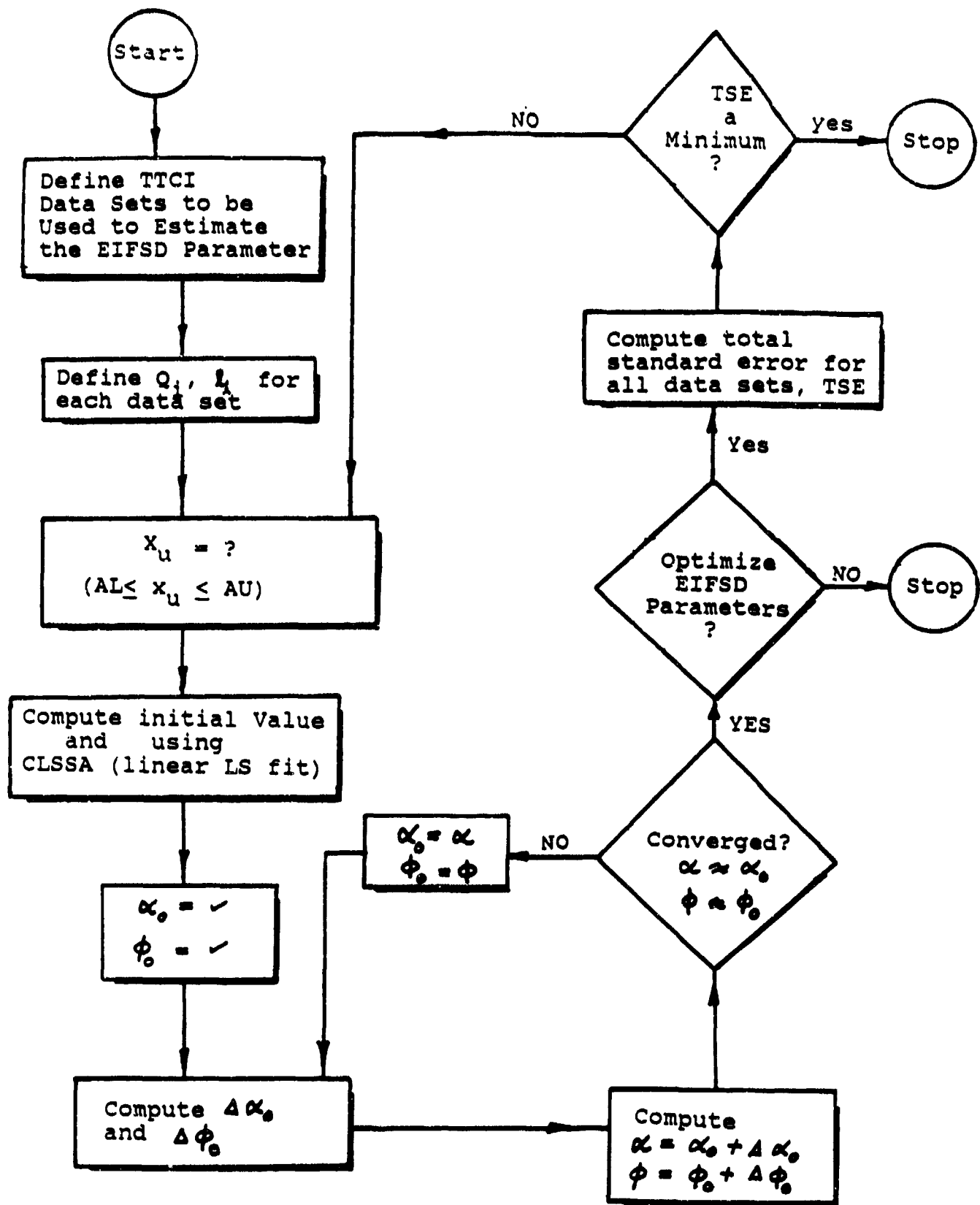


Figure F.3. Schematic for Estimating/Optimizing the EIFSD Parameters Based on a Non-Linear Approach.

## APPENDIX G

### FRACTOGRAPHIC DATA PROCESSING METHODS

#### G.1 INTRODUCTION

Data processing methods are needed to prepare raw fractographic data for durability analyses because fractographic data for each fatigue crack in the data set may not be homogeneous. For example, the fractographic data for a given data set may not cover the same crack size range. The minimum and maximum crack size readings typically vary from crack-to-crack (e.g., see Fig. G.1). Fractographic data processing methods are needed to: (1) obtain a homogeneous fractographic data set and (2) extrapolate and/or interpolate fractographic results to define TTCIs for a selected reference crack size ( $x_1$ ). Methods are described and discussed in this section for processing raw fractographic data for durability analyses.

#### G.2 RAW FRACTOGRAPHIC DATA

Raw fractographic data situations are described in Fig. G.1. Various aspects are discussed in this section.

In Fig. G.1 the fractographic results for six different fatigue cracks are depicted. The fractographic crack size range for determining the "pooled Q" value is denoted by AL-AU. The reference crack size for TTCIs is denoted by  $x_1$ . The following observations are based on Fig. G.1: (1) fractographic results for crack ① cover the AL-AU range, (2) cracks ② through ④ have fractographic data in the AL-AU range but the minimum and/or maximum crack size reading do not cover the AL-AU range, (3) there is no fractographic

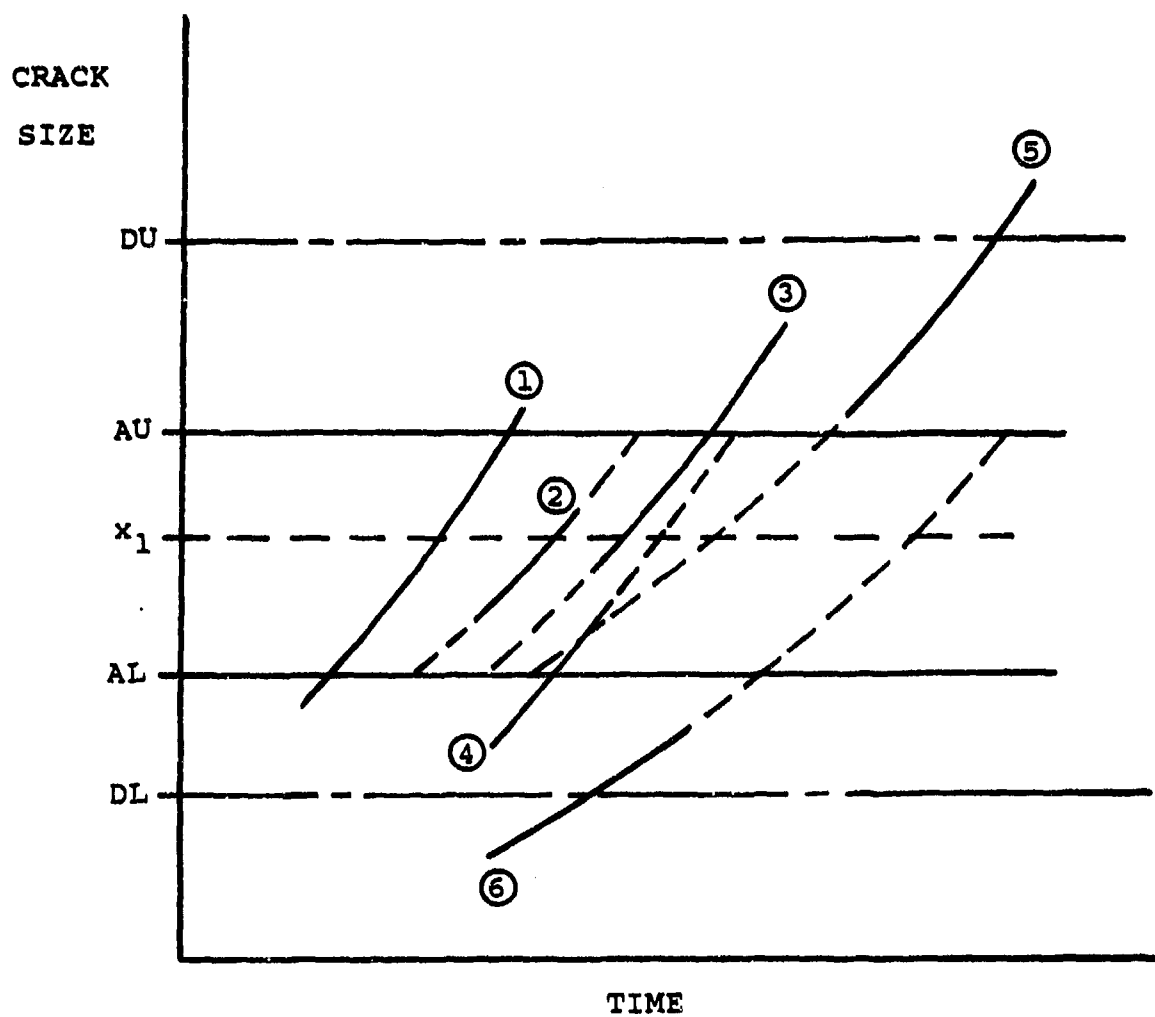


Figure G.1. Raw Fractographic Data May Need to be Processed.

data in the AL-AU range for cracks ⑤ and ⑥, (4) the TTCIs for cracks ① and ② can be obtained by interpolation, and (5) the fractographic results for cracks ③ through ⑥ must be extrapolated to determine the TTCIs at  $x_1$ . In Fig. G.1, dashed lines indicate fractographic data extrapolations to the AL and/or AU boundary.

The "pooled Q" value for a given data set and the EIFSD parameters are identified for a given AL-AU crack size range (e.g., 0.01"-0.05"). In some cases there may be no fractographic data in the AL-AU range. Such cases are illustrated by cracks ⑤ and ⑥ in Fig. G.1. To utilize the fractographic data for cracks ⑤ and ⑥, a default crack size range is selected, DL-DU, so that the fractographic results within the DL-DU range can be extrapolated for applications in the desired AL-AU range.

### G.3 EXTRAPOLATION METHODS

A reasonable method is described in this section for extrapolating fractographic data. This method is based on the crack size-time relationship given in Eq. C-3 and the following least squares fit procedure. Other extrapolation methods could also be used. For example, refer to extrapolation methods reviewed by Penny and Marriott [60].

In Fig. G.2 the fractographic data (i.e.,  $a(t)$  vs.  $t$  results) for a given fatigue crack are denoted by solid circles. Suppose the TTCI value,  $t_1$ , is beyond the range of the fractographic data an extrapolation is required to determine  $t_1$  at  $x_1$ .

The generalized crack size-time relationship given in Eq. C-3 can be transformed into a linear least square fit form as shown in Eq. C-5. In this case,  $Q_1$ , is the deterministic crack growth rate parameter for a given fatigue crack.

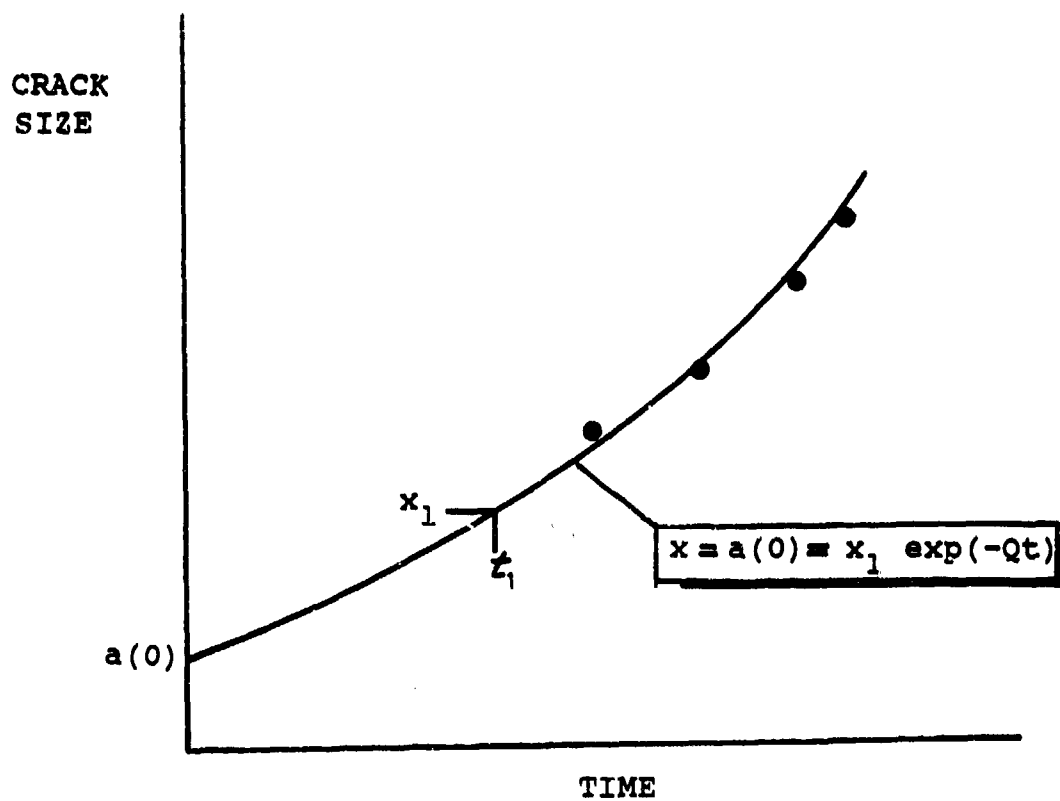


Figure G.2. Extrapolation of Fractographic Data Using Generalized Crack Size-Time Relationship.

The other terms in Eq. C-5 have already been defined. A least squares fit procedure can be used to determine the constants  $Q_1$  and  $a(0)$  in Eq. C-5. Expressions for  $Q_1$  and  $a(0)$  are given in Eqs. C-6 and G-1, respectively. The sums in Eq. G-1 can be determined using the fractographic data.

$$EIFS = a(0) = \exp \left\{ \frac{\sum_{i=1}^N \ln a(t_i) - Q_1 \sum_{i=1}^N t_i}{N} \right\} \quad (G-1)$$

Once the constants  $Q_1$  and  $a(0)$  have been determined, the TICI,  $t_1$ , can be determined from the generalized crack size-time relationship expressed in terms of  $t_1$ . Such an expression for  $t_1$  is given in Eq. G-2.

$$t_1 = \left( \frac{1}{Q_1} \right) \ln \left[ x_1 / a(0) \right] \quad (G-2)$$

Equation G-2 can be expressed in terms of  $x_1$  as shown in Eq. G-3. This equation can be used to extrapolate the fractographic results to a selected  $t_1$  beyond the range of the fractographic data.

$$x_1 = a(0) \exp(Q_1 t_1) \quad (G-3)$$

The  $Q_1$  value in Eq. C-5 can also be used in the crack growth rate model, given in Eq. 1 where  $b = 1$ ), to estimate the crack growth rates at  $a(t)$ ,  $t$  points beyond the range of the fractographic data.

#### G-4 DISCUSSION

Fractographic results may be limited or non-existent for the desired durability analysis application. To effectively use available fractographic data, reasonable data processing procedures should be used to obtain a homogeneous data set. The fractographic data should be processed in a consistent

and reasonable manner. In any case, beware of extrapolating far beyond the limits of the actual fractographic data. Computer software is available for screening and processing the raw fractographic data [6].

## APPENDIX H

### INITIAL FLAW SIZE

The following are discussed in this section: (1) initial flaw size needs for durability analysis, (2) the meaning and limitations of EIFS in a durability analysis, (3) EIFSD functions with an "open" and "closed" upper tail, (4) how initial flaw sizes can be determined from the EIFSD function, and (5) expressions for initial flaw sizes for four EIFSD functions. The size of the initial flaw depends on the EIFSD function used, the desired probability of crack exceedance and the EIFS upper bound limit,  $x_u$ .

#### H.1 DURABILITY ANALYSIS NEEDS

Functional impairment design requirements for excessive cracking and for fuel leakage/ligament breakage can be evaluated using the entire EIFSD for a population of structural details. In this case, the entire EIFSD is grown forward to determine the probability of crack exceedance and/or functional impairment.

Functional impairment due to fuel leakage and ligament breakage can also be evaluated using an initial flaw size obtained from the EIFSD. In this case, the durability analysis is similar to the typical damage tolerance analysis. Excessive cracking is concerned with relatively small fatigue cracks for the entire population of structural details. Whereas, fuel leakage and ligament breakage are generally concerned with selected critical control points within a structure where this type of functional impairment may occur. In this case, large through-the-thickness type cracks (e.g., 0.50" - 0.75") may cause functional impairment.



There are subtle differences between using a single initial flaw size and the entire EIFSD for durability analysis. When a single initial flaw size,  $x_1$ , is used the initial flaw is obtained from the EIFSD and it corresponds to a selected crack exceedance probability (e.g., 0.001). Suppose the initial flaw size,  $x_1$ , is grown forward to a crack size  $x_1$  at time  $t$ . Then, it can be said in this case that the probability of exceeding the crack size  $x_1$  is  $p(i, \tau) = 0.0001$ , which is the same probability associated with the selected initial flaw size. On the other hand, suppose the entire EIFSD is to be grown forward to time  $t$  the same way the single initial flaw size  $x_1$  was grown. In this case, the probability of exceeding a crack size  $x_1$  at time  $t$  would be the same as that based on the single initial flaw growth.

The entire EIFSD should be used to evaluate functional impairment due to excessive cracking. In this case, the probability of exceeding a specified crack size  $x_1$  at time  $t$  is the question. Since  $p(i, \tau)$  varies depending on the  $x_1$  value, a single initial flaw size  $x_1$  cannot be used to make crack exceedance predictions for different  $x_1$  values.

## H.2 MEANING AND LIMITATIONS OF EIFSS

The meaning and limitations of an initial flaw obtained from the EIFSD are as follows:

1. The initial flaw is artificial - not a real flaw, such as a scratch, burr or void in a fastener hole. Therefore, the size of artificial initial flaws are not necessarily comparable with minimum fractographic crack sizes obtained from tear-down inspection results. Furthermore, an artificial initial flaw cannot be verified by NDI.

2. An artificial flaw defines a characteristic crack size that applies to different crack shapes and geometries.

3. An artificial initial flaw depends on the: (1) fractographic crack size range used, AL-AU, (2) crack growth rate model used for back-extrapolations (DCGA or SCGA), (3) EIFSD function used and (4) upper bounds limits ( $x_u$ ). These factors define the "basis" for the EIFS.

4. All artificial initial flaws or EIFSs must be considered in the same context that such flaws were defined. For example, an EIFS must be grown forward for crack growth predictions in the same manner the EIFS was determined. In other words, when an EIFS is grown forward in a crack growth analysis the EIFS is subject to the same limitations and conditions used to define the EIFSs in the first place. This is a very important point. For example, once an artificial initial flaw has been determined for a given crack exceedance probability, this initial flaw cannot be indiscriminately grown forward to any crack size using a LEFM crack growth program [e.g., 38, 39] without considering the "basis" for the EIFSs.

5. EIFSs and EIFSDs are defined using an "engineering approach." This approach does not reflect a mechanistic-based characterization of the complex microcracking process. Durability "analysis tools" are needed for design purposes. The "engineering approach" satisfies the needs for durability analysis during the design stage.

### H.3 COMPARISON OF EIFSD FUNCTIONS

Two types of EIFSD functions will be discussed: (1) distribution function with closed upper tail (e.g., Weibull compatible and lognormal compatible) and (2) distribution

function with open upper tail (e.g., two-parameter Weibull and lognormal). The two types of EIFSD functions are shown in Fig. H.1.

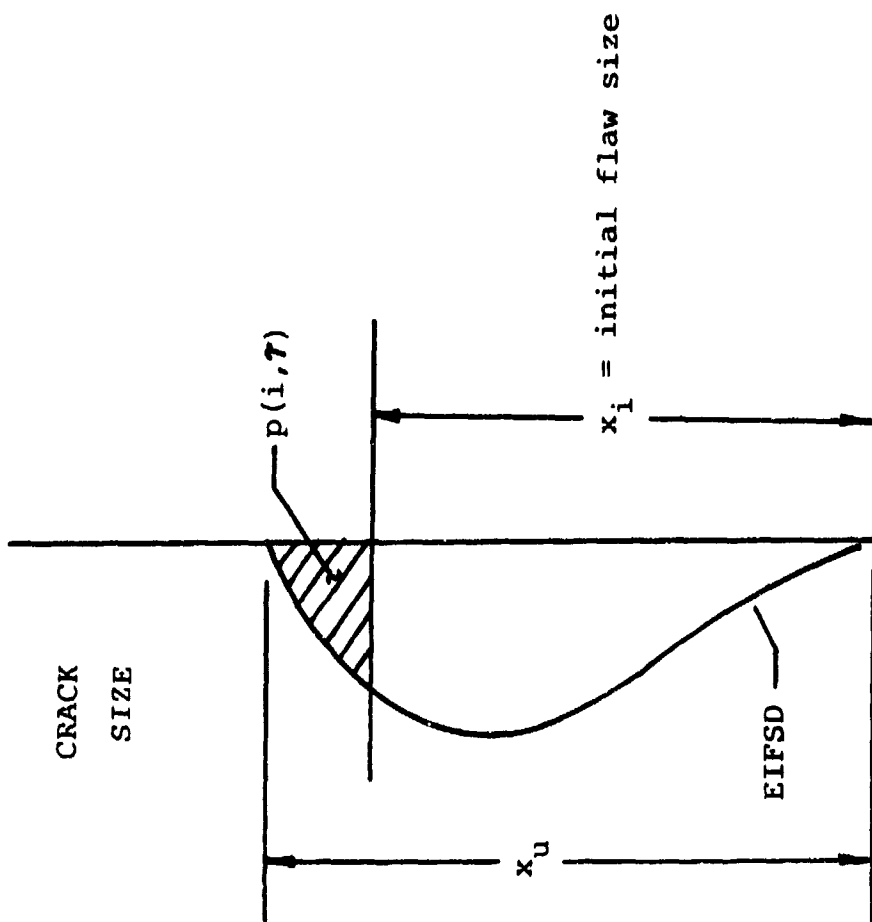
### H.3.1 EIFSD Function with Closed Upper Tail

A compatible type EIFSD function is derived using a reasonable cumulative distribution function for TTCI and an appropriate deterministic crack growth law. In this case, the resulting EIFSD is said to be "compatible with the TTCID." As shown in Fig. H.1(a), the compatible type EIFSD function has a closed upper tail. This means that the largest initial flaw size in the EIFSD is equal to the EIFS upper bound limit,  $x_u$ . In most cases, the initial flaw size of most interest lies in the extreme upper tail of the EIFSD function. For example, suppose the initial flaw size,  $x_i$ , corresponding to a probability of crack exceedance  $p(i, \tau)$ , of 1 out of 10000 (0.0001) is desired. In Fig. H.1(a),  $p(i, \tau)$  is equal to the cross-hatched area under the EIFSD. It is seen in Fig. H.1(a) that the initial flaw size,  $x_i$ , approaches  $x_u$  as a limit at the  $p(i, \tau)$  approaches 0. Thus,  $x_u$  has a strong influence on the extreme initial flaw size desired for durability analysis.

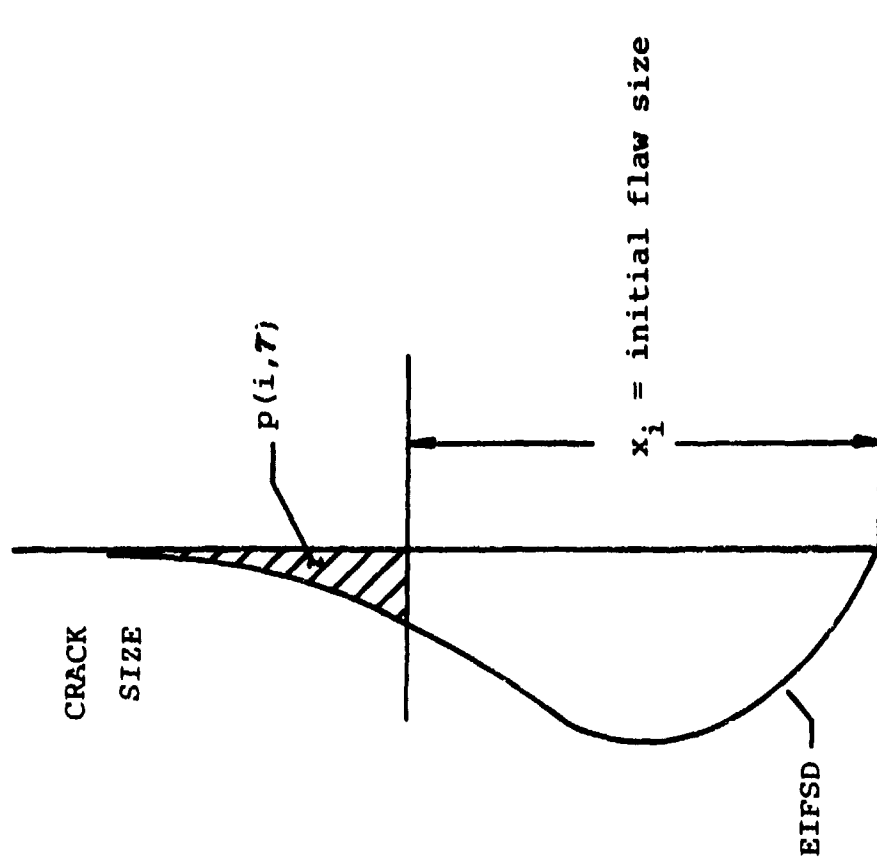
### H.3.2 EIFSD Function with Open Upper Tail

In some cases, the user may want to assume an EIFSD function for representing the IFQ which has an open upper tail as shown in Fig. H.1(b). For example, the two-parameter Weibull and the lognormal distribution functions have this characteristic.

A compatible-type EIFSD function is preferred because it has a sounder basis than the non-compatible type. For example, the compatible-type EIFSD function is based on a physically-meaningful TTCID function which models the fatigue



(a) Distribution Function With Closed Upper End (e.g., Weibull Compatible)



(b) Distribution Function With Open Upper Tail (e.g., Two-Parameter Weibull & Lognormal)

Figure H.1. Comparison of Distribution Functions with Closed and Open Upper Tails.

wearout process and trends. On the other hand, if the EIFSD function is simply assumed without regard to the fatigue wearout process, the resulting EIFSD may not be physically meaningful.

In Fig. H.1 the "closed upper tail" EIFSD is conceptually compared with the "open upper tail" EIFSD. The "open upper tail" type EIFSD has three general characteristics: (1) since there is no upper bound limit on the EIFSs, infinite initial flaw sizes are possible and (2) the shape of the EIFSD is typically skewed downward whereas the compatible-type EIFSD is skewed upward and (3) when the EIFSD is grown forward to predict the cumulation distribution of TCI,  $F_T(t)$ , a truncated TTCID is required to assure that all TTCIs will be positive (e.g., see Fig. H.2).

### H.3.3 Initial Flaw Size Expressions

Initial flaw size expressions for four different functions are summarized in Table H.1. Once the EIFSD parameters have been determined for a given EIFSD function, an initial flaw size can be determined for a given probability of crack exceedance. For example, an initial flaw size,  $x_1$ , corresponding to a probability of crack exceedance,  $p(1, T)$ , for 1 out of 10000 (or 0.0001) can be determined. Further considerations are required to define the initial flaw size for a given probability and confidence level.

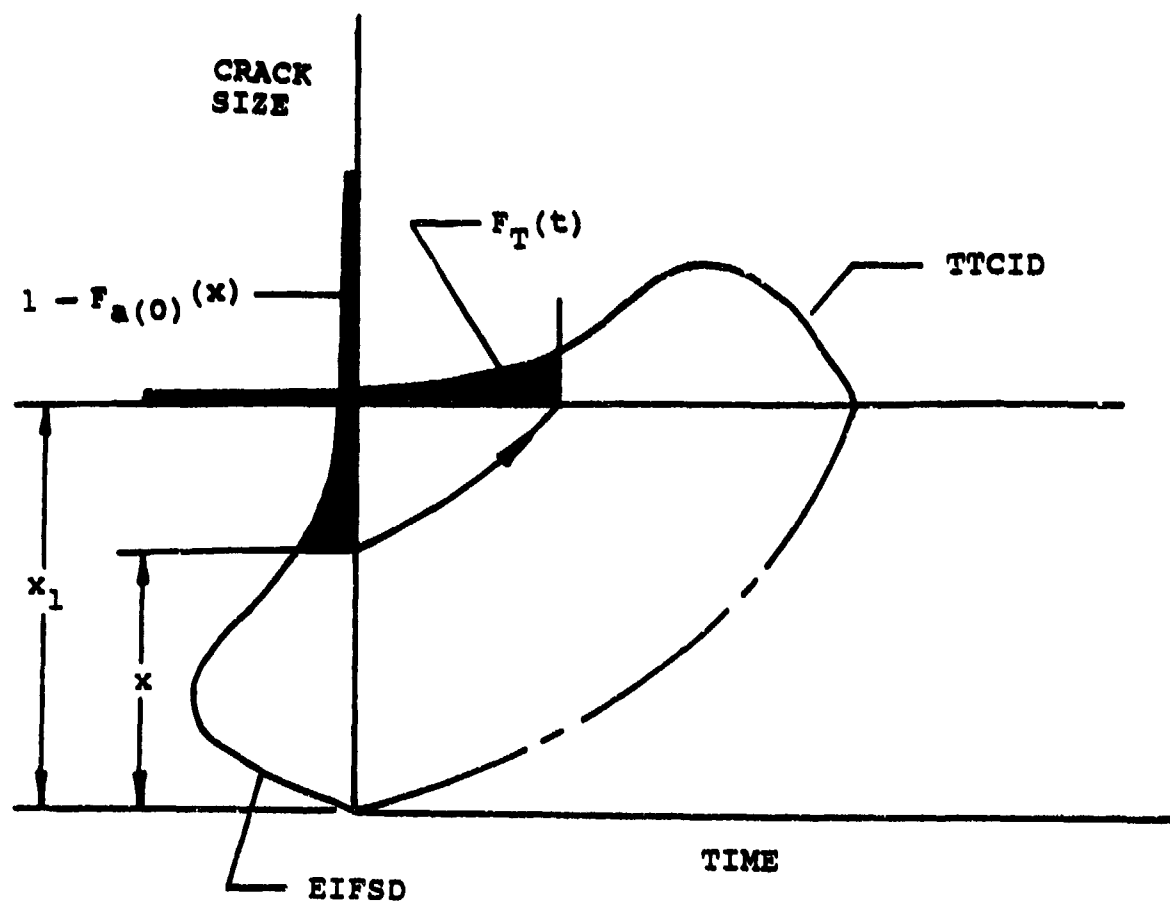


Figure H.2. Transformation of Open Upper Tail Type EIFSD to Obtain Cumulative Distribution of TTCI.

TABLE H.1. Summary of Expressions for Initial Flaw Size for Four Different EIFSD Functions.

| EIFSD Function        | Cumulative Distribution of EIFS, $F_{a(0)}(x)$  | Initial Flaw Size Expression   |
|-----------------------|---|--|
| Weibull Compatible    | $F_{a(0)}(x) = \exp \left\{ - \left  \frac{\ln(x/x_u)}{\phi} \right ^a \right\}$      | $x = \exp \left\{ \ln x_u - \phi \left[ -\ln F_{a(0)}(x) \right]^{1/a} \right\}$                               |
| Lognormal Compatible  | $F_{a(0)}(x) = 1 - \phi \left\{ \frac{\ln \ln(x/x_u) - \ln Q}{\sigma} \right\} - \mu$ | $x = \exp \left\{ \exp \left[ \ln \ln x_u - \ln Q - \mu - \sigma \phi^{-1} [1 - F_{a(0)}(x)] \right] \right\}$ |
| Two-Parameter Weibull | $F_{a(0)}(x) = 1 - \exp \left\{ - \left( \frac{x}{x'_0} \right)^a \right\}$           | $x = \beta \left\{ -\ln [1 - F_{a(0)}(x)] \right\}^{1/a}$  |
| Lognormal             | $F_{a(0)}(x) = \Phi \left\{ \frac{\ln x - u}{\sigma} \right\}$                        | $x = \exp \left\{ \mu + \sigma \phi^{-1} [F_{a(0)}(x)] \right\}$   |

